Name____ Date_______________________ Period_____

Worksheet 6.3—Volumes
Show all work. No calculator unless stated.

Multiple Choice

1. (Calculator Permitted) The base of a solid *S* is the region enclosed by the graph of $y = \ln x$, the line $x = e$, and the *x*-axis. If the cross sections of *S* perpendicular to the *x*-axis are squares, which of the following gives the best approximation of the volume of *S*?
(A) 0.718 (B) 1.718 (C) 2.718 (D) 3.171 (E) 7.388

2. (Calculator Permitted) Let *R* be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the *x*-axis, and the *y*-axis. Which of the following gives the best approximation of the volume of the solid generated when *R* is revolved about the *x*-axis?
(A) 60.3 (B) 115.2 (C) 225.4 (D) 319.7 (E) 361.9

3. Let R be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the y-axis.

(A) 64π (B) 128π (C) 256π (D) 360 (E) 512

4. Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^{x}$, and $x = 1$. Which of the following gives the volume of the solid generated when R is revolved about the x-axis?

(A) $\int_0^1 \left(e^x - e^{-x}\right) dx$ (B) $\int_0^1 \left(e^{2x} - e^{-2x}\right) dx$ (C) $\int_0^1 \left(e^x - e^{-x}\right)^2 dx$

(D) $\pi\int_0^1 \left(e^{2x} - e^{-2x}\right) dx$ (E) $\pi\int_0^1 \left(e^x - e^{-x}\right)^2 dx$

5. (Calculator Permitted) The base of a solid is the region in the first quadrant bounded by the x-axis, the graph of $y = \sin^{-1} x$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume?

(A) 0.117 (B) 0.285 (C) 0.467 (D) 0.571 (E) 1.571

6. Let R be the region in the first quadrant bounded by the graph of $y = 3x - x^2$ and the x-axis. A solid is generated when R is revolved about the vertical line $x = -1$. Set up, but do not evaluate, the definite integral that gives the volume of this solid.

(A) $\int_0^3 2\pi(x+1)\left(3x - x^2\right)dx$ (B) $\int_{-1}^3 2\pi(x+1)\left(3x - x^2\right)dx$ (C) $\int_0^3 2\pi(x)\left(3x - x^2\right)dx$

(D) $\int_0^3 2\pi\left(3x - x^2\right)^2 dx$ (E) $\int_0^3 \left(3x - x^2\right)dx$

Free Response

7. (Calculator Permitted) Let *R* be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the *y*-axis.
   (a) Find the area of *R*.

   (b) Find the volume of the solid generated when *R* is revolved about the line $y = -1$.

   (c) The region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.

8. (Calculator Permitted) The base of the volume of a solid is the region bounded by the curve $y = 2 + \sin x$, the x-axis, $x = 0$, and $x = \frac{3\pi}{2}$. Find the volume of the solids whose cross sections perpendicular to the x-axis are the following:

(a) Squares

(b) Rectangles whose height is 3 times the base

(c) Equilateral triangles

(d) Isosceles right triangles with a leg on the base

(e) Isosceles triangles with hypotenuse on the base

(f) Semi-circles

(g) Quarter-circles

9. (Calculator Permitted) Let R be the region bounded by the curves $y = x^2 + 1$ and $y = x$ for $0 \le x \le 1$. Showing all integral set-ups, find the volume of the solid obtained by rotating the region R about the

(a) x-axis

(b) y-axis

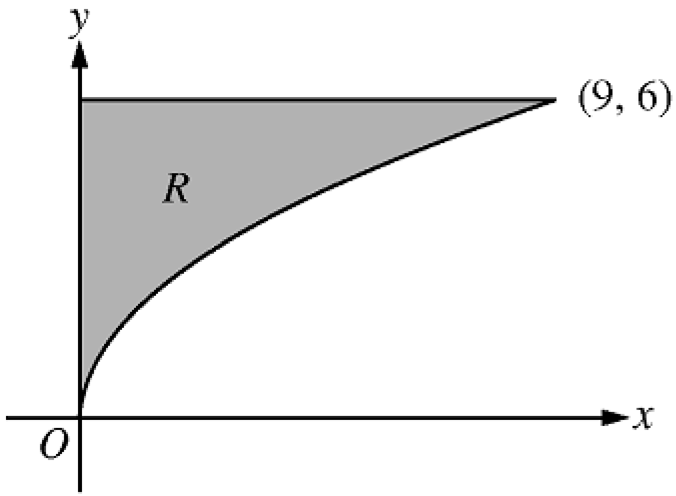
(c) the line $x = 2$

(d) the line $x = -1$

(e) the line $y = -1$

(f) the line $y = 3$

10. (AP 2010-4) Let *R* be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the *y*-axis, as shown in the figure below.

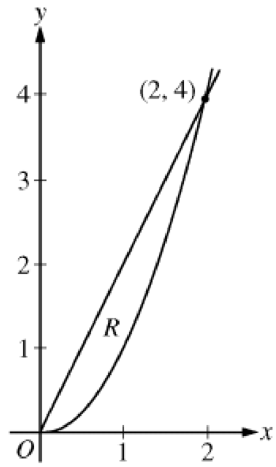


(a) Find the area of *R*.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when *R* is rotated about the horizontal line $y = 7$.

(c) Region *R* is the base of a solid. For each *y*, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the ***y*-axis** is a rectangle whose height is 3 times the length of its base in region *R*. Write, but do not evaluate, and integral expression that gives the volume of this solid.

11. (AP 2009-4) Let *R* be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure.

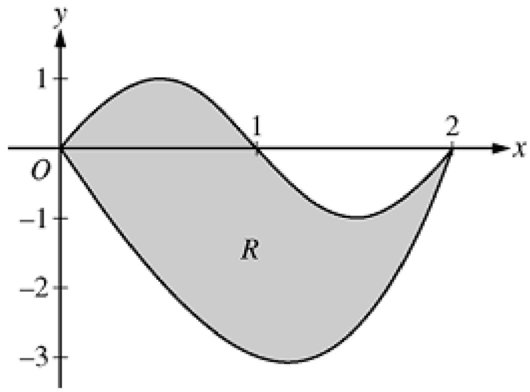


(a) Find the area of *R*.

(b) The region *R* is the base of the solid. For this solid, at each *x*, the cross section perpendicular to the *x*-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base *R*. For this solid, the cross sections perpendicular to the *y*-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

12. (AP 2008-1) (Calculator Permitted) Let *R* be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure.



(a) Find the area of *R*.

(b) The horizontal line $y = -2$ splits the region *R* into two parts. Write, but do not evaluate, and integral expression for the area of the part of *R* that is below this horizontal line.

(c) The region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume of this solid.

(d) The region *R* models the surface of a small pond. At all points in *R* at a distance *x* from the *y*-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

13. (AP 2007-1) (Calculator Permitted) Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections, perpendicular to the x-axis, are semicircles. Find the volume of this solid.

14. (AP 2002-1) (Calculator Permitted) Let *f* and *g* be the functions given by $f(x)=e^x$ and $g(x)=\ln x$.

(a) Find the area of the region enclosed by the graphs of *f* and *g* between $x=\frac{1}{2}$ and $x=1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of *f* and *g* between $x=\frac{1}{2}$ and $x=1$ is revolved about the line $y=4$.

(c) Let *h* be the function given by $h(x)=f(x)-g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2}\le x\le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2}\le x\le 1$. Show the analysis that leads to your answer.