Worksheet 6.3—Volumes
Show all work. No calculator unless stated.

Multiple Choice

1. (Calculator Permitted) The base of a solid $S$ is the region enclosed by the graph of $y = \ln x$, the line $x = e$, and the $x$-axis. If the cross sections of $S$ perpendicular to the $x$-axis are squares, which of the following gives the best approximation of the volume of $S$?
   (A) 0.718  (B) 1.718  (C) 2.718  (D) 3.171  (E) 7.388

2. (Calculator Permitted) Let $R$ be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the $x$-axis, and the $y$-axis. Which of the following gives the best approximation of the volume of the solid generated when $R$ is revolved about the $x$-axis?
   (A) 60.3  (B) 115.2  (C) 225.4  (D) 319.7  (E) 361.9
3. Let $R$ be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis. Which of the following gives the best approximation of the volume of the solid generated when $R$ is revolved about the y-axis.

(A) $64\pi$  (B) $128\pi$  (C) $256\pi$  (D) $360$  (E) $512$

4. Let $R$ be the region enclosed by the graphs of $y = e^{-x}$, $y = e^x$, and $x = 1$. Which of the following gives the volume of the solid generated when $R$ is revolved about the x-axis?

(A) $\int_0^1 (e^x - e^{-x}) \, dx$  (B) $\int_0^1 (e^{2x} - e^{-2x}) \, dx$  (C) $\int_0^1 (e^x - e^{-x})^2 \, dx$

(D) $\pi \int_0^1 (e^{2x} - e^{-2x}) \, dx$  (E) $\pi \int_0^1 (e^x - e^{-x})^2 \, dx$
5. (Calculator Permitted) The base of a solid is the region in the first quadrant bounded by the \( x \)-axis, the graph of \( y = \sin^{-1} x \), and the vertical line \( x = 1 \). For this solid, each cross section perpendicular to the \( x \)-axis is a square. What is the volume?
(A) 0.117       (B) 0.285       (C) 0.467       (D) 0.571       (E) 1.571

6. Let \( R \) be the region in the first quadrant bounded by the graph of \( y = 3x - x^2 \) and the \( x \)-axis. A solid is generated when \( R \) is revolved about the vertical line \( x = -1 \). Set up, but do not evaluate, the definite integral that gives the volume of this solid.
(A) \( \int_{0}^{3} 2\pi (x+1)(3x-x^2) \, dx \)  
(B) \( \int_{-1}^{3} 2\pi (x+1)(3x-x^2) \, dx \)  
(C) \( \int_{0}^{3} 2\pi (x)(3x-x^2) \, dx \)  
(D) \( \int_{0}^{3} 2\pi (3x-x^2)^2 \, dx \)  
(E) \( \int_{0}^{3} (3x-x^2) \, dx \)
Free Response

7. (Calculator Permitted) Let $R$ be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the $y$-axis.
   (a) Find the area of $R$.

   (b) Find the volume of the solid generated when $R$ is revolved about the line $y = -1$.

   (c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.
8. (Calculator Permitted) The base of the volume of a solid is the region bounded by the curve 

\[ y = 2 + \sin x, \quad \text{x-axis, } x = 0, \quad \text{and } x = \frac{3\pi}{2}. \]

Find the volume of the solids whose cross sections perpendicular to the x-axis are the following:

(a) Squares

(b) Rectangles whose height is 3 times the base

(c) Equilateral triangles

(d) Isosceles right triangles with a leg on the base

(e) Isosceles triangles with hypotenuse on the base

(f) Semi-circles

(g) Quarter-circles
9. (Calculator Permitted) Let $R$ be the region bounded by the curves $y = x^2 + 1$ and $y = x$ for $0 \leq x \leq 1$. Showing all integral set-ups, find the volume of the solid obtained by rotating the region $R$ about the

(a) $x$-axis  
(b) $y$-axis  
(c) the line $x = 2$

(d) the line $x = -1$  
(e) the line $y = -1$  
(f) the line $y = 3$
10. (AP 2010-4) Let \( R \) be the region in the first quadrant bounded by the graph of \( y = 2\sqrt{x} \), the horizontal line \( y = 6 \), and the y-axis, as shown in the figure below.

(a) Find the area of \( R \).

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when \( R \) is rotated about the horizontal line \( y = 7 \).

(c) Region \( R \) is the base of a solid. For each \( y \), where \( 0 \leq y \leq 6 \), the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region \( R \). Write, but do not evaluate, and integral expression that gives the volume of this solid.
11. (AP 2009-4) Let $R$ be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure.

(a) Find the area of $R$.

(b) The region $R$ is the base of the solid. For this solid, at each $x$, the cross section perpendicular to the $x$-axis has area $A(x) = \sin \left( \frac{\pi}{2} x \right)$. Find the volume of the solid.

(c) Another solid has the same base $R$. For this solid, the cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.
12. (AP 2008-1) (Calculator Permitted) Let $R$ be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure.

(a) Find the area of $R$.

(b) The horizontal line $y = -2$ splits the region $R$ into two parts. Write, but do not evaluate, and integral expression for the area of the part of $R$ that is below this horizontal line.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Find the volume of this solid.

(d) The region $R$ models the surface of a small pond. At all points in $R$ at a distance $x$ from the $y$-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.
13. (AP 2007-1) (Calculator Permitted) Let $R$ be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the $x$-axis.

(c) The region $R$ is the base of a solid. For this solid, the cross sections, perpendicular to the $x$-axis, are semicircles. Find the volume of this solid.
14. (AP 2002-1) (Calculator Permitted) Let \( f(x) = e^x \) and \( g(x) = \ln x \).

(a) Find the area of the region enclosed by the graphs of \( f \) and \( g \) between \( x = \frac{1}{2} \) and \( x = 1 \).

(b) Find the volume of the solid generated when the region enclosed by the graphs of \( f \) and \( g \) between \( x = \frac{1}{2} \) and \( x = 1 \) is revolved about the line \( y = 4 \).

(c) Let \( h \) be the function given by \( h(x) = f(x) - g(x) \). Find the absolute minimum value of \( h(x) \) on the closed interval \( \frac{1}{2} \leq x \leq 1 \), and find the absolute maximum value of \( h(x) \) on the closed interval \( \frac{1}{2} \leq x \leq 1 \). Show the analysis that leads to your answer.