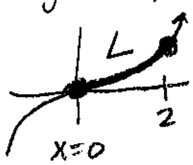
WS 8.4 – Arc Length KEY CALCULUS MAXIMUS

① $y = x^3,\ x \in [0,2]$

$y' = 3x^2$



$L = \int_0^2 \sqrt{1 + (3x^2)^2}\, dx$

$= \boxed{\int_0^2 \sqrt{1 + 9x^4}\, dx}$ $\boxed{E}$

② $L = \int_1^4 \sqrt{1 + 9x^4}\, dx$

So $(f'(x))^2 = 9x^4$

$f'(x) = 3x^2$ or $-3x^2 = f'(x)$

$f(x) = x^3 + C$ or $f(x) = -x^3 + C$

at $(1,6)$: $6 = 1^3 + C$ or $6 = -1 + C$

$C = 5$ $\quad$ $C = 7$

So $\boxed{f(x) = x^3 + 5}$ or $f(x) = -x^3 + 7$

$\boxed{B}$

③ $y = \cos(2x),\ x \in [0, \frac{\pi}{4}]$

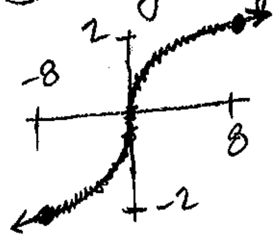
$y' = -2\sin(2x)$

$L = \int_0^{\pi/4} \sqrt{1 + (-2\sin(2x))^2}\, dx$

$= \int_0^{\pi/4} \sqrt{1 + 4\sin^2(2x)}\, dx$

$= 1.317$ or $\boxed{1.318}$ $\boxed{D}$

④ $x = y^3 \Leftrightarrow y = \sqrt[3]{x}$, $y = -2$ to $y = 2$ or $x = -8$ to 8



* since y' is undefined at $x = 0$ (vertical tangent line), we will find the same length by going through $y^{-1} = g(x)$ on an equivalent interval.

$g(x) = x^3$ on $x \in [-2, 2]$, $g'(x) = 3x^2$

$L = \int_{-2}^{2} \sqrt{1 + (3x^2)^2}\, dx = \int_{-2}^{2} \sqrt{1 + 9x^4}\, dx$

Now x, dx can be ANY variables, so

$\int_{-2}^{2} \sqrt{1 + 9x^4}\, dx = \int_{-2}^{2} \sqrt{1 + 9p^4}\, dp = \boxed{\int_{-2}^{2} \sqrt{1 + 9y^4}\, dy}$ $\boxed{C}$

⑤ $y = \frac{2}{3}x^{3/2},\ x \in [0, 8]$

$y' = x^{1/2}$

$L = \int_0^8 \sqrt{1 + (x^{1/2})^2}\, dx$

$= \int_0^8 \sqrt{1 + x}\, dx$

$= \int_0^8 (1 + x)^{1/2}\, dx$

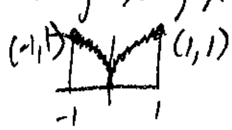
$= \frac{2}{3}(1 + x)^{3/2} \Big|_0^8$

$= \frac{2}{3}\left[(9^{3/2}) - (1^{3/2})\right]$

$= \frac{2}{3}[27 - 1]$

$= \boxed{\frac{52}{3}}$ $\boxed{B}$

⑥ $y = x^{2/3},\ x \in [-1, 1]$, y' is undefined at $x = 0$



using symmetry and going through the inverse

Let $y^{-1} = g(x)$

$g(x) = x^{3/2}$

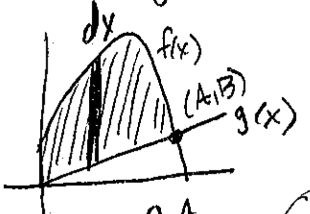
$g'(x) = \frac{3}{2}x^{1/2}$

$L = 2\int_0^1 \sqrt{1 + (\frac{3}{2}x^{1/2})^2}\, dx$

$= 2\int_0^1 \sqrt{1 + \frac{9}{4}x}\, dx$

$= \boxed{2\int_0^1 \sqrt{1 + \frac{9}{4}y}\, dy}$ $\boxed{A}$

7) $x=0,\ y=f(x)=4x-x^3+1,\ y=g(x)=\frac{3}{4}x$



intersection

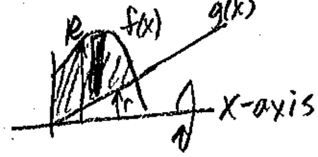
$f(x)=g(x)$

$\boxed{\text{at } (x,y)=(A,B)=(1.940,\ 1.455)}$

(a) $\text{Area}=\int_0^A \left(f(x)-g(x)\right)dx$

$=\boxed{4.514 \text{ or } 4.515}$

(b)



perpenWASHular Method

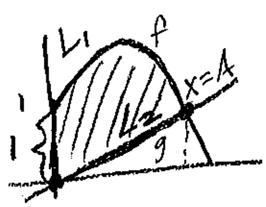
$R(x)=f(x)-0=f(x)$

$r(x)=g(x)-0=g(x)$

$\text{Volume}=\pi\int_0^A\left[(f(x))^2-(g(x))^2\right]dx$

$=\boxed{57.463}$

(c) Perimeter $=P$



$f'(x)=4-3x^2$

$g'(x)=\frac{3}{4}$

$P=1+L_1+L_2$

$$\boxed{P=1+\int_0^A\sqrt{1+(4-3x^2)^2}\,dx+\int_0^A\sqrt{1+\left(\tfrac{3}{4}\right)^2}\,dx}$$

or, since L_2 is a line segment, the distance formula can be used

$$P=1+\sqrt{B^2+A^2}+\int_0^A\sqrt{1+(4-3x^2)^2}\,dx$$