

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

**Worksheet 6.4—Arc Length**

Show all work. No calculator unless stated.

**Multiple Choice**1. ('88 BC) The length of the curve  $y = x^3$  from  $x = 0$  to  $x = 2$  is given by

$$(A) \int_0^2 \sqrt{1+x^6} dx \quad (B) \int_0^2 \sqrt{1+3x^2} dx \quad (C) \pi \int_0^2 \sqrt{1+9x^4} dx$$

$$(D) 2\pi \int_0^2 \sqrt{1+9x^4} dx \quad (E) \int_0^2 \sqrt{1+9x^4} dx$$

2. ('03 BC) The length of a curve from  $x = 1$  to  $x = 4$  is given by  $\int_1^4 \sqrt{1+9x^4} dx$ . If the curve contains the point  $(1,6)$ , which of the following could be an equation for this curve?

$$(A) y = 3 + 3x^2 \quad (B) y = 5 + x^3 \quad (C) y = 6 + x^3$$

$$(D) y = 6 - x^3 \quad (E) y = \frac{16}{5} + x + \frac{9}{5}x^5$$

3. (Calculator Permitted) Which of the following gives the best approximation of the length of the arc of

$$y = \cos(2x) \text{ from } x = 0 \text{ to } x = \frac{\pi}{4}?$$

- (A) 0.785    (B) 0.955    (C) 1.0    (D) 1.318    (E) 1.977

4. Which of the following gives the length of the graph of  $x = y^3$  from  $y = -2$  to  $y = 2$ ?

- (A)  $\int_{-2}^2 (1 + y^6) dy$     (B)  $\int_{-2}^2 \sqrt{1 + y^6} dy$     (C)  $\int_{-2}^2 \sqrt{1 + 9y^4} dy$     (D)  $\int_{-2}^2 \sqrt{1 + x^2} dx$     (E)  $\int_{-2}^2 \sqrt{1 + x^4} dx$

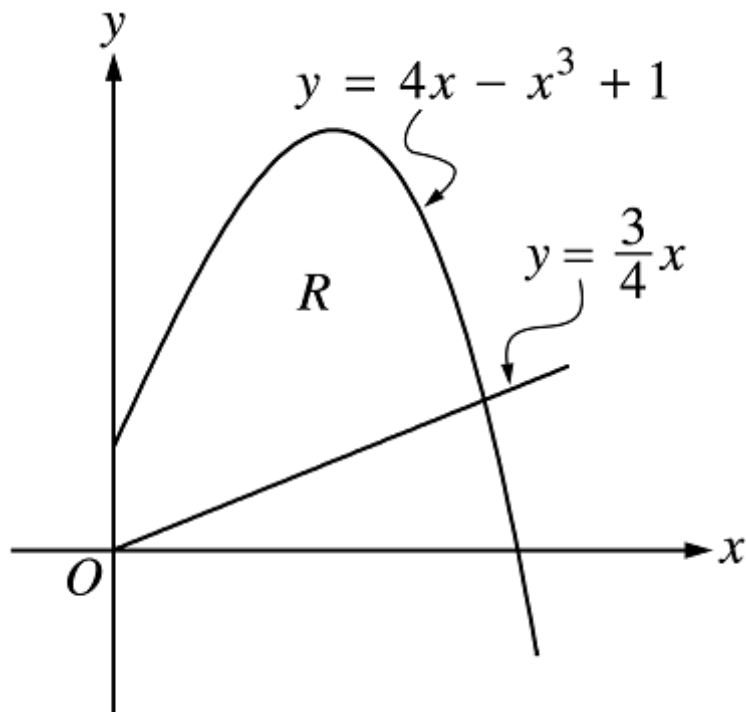
5. Find the length of the curve described by  $y = \frac{2}{3}x^{3/2}$  from  $x = 0$  to  $x = 8$ .

- (A)  $\frac{26}{3}$     (B)  $\frac{52}{3}$     (C)  $\frac{512\sqrt{2}}{15}$     (D)  $\frac{512\sqrt{2}}{15} + 8$     (E) 96

6. Which of the following expressions should be used to find the length of the curve  $y = x^{2/3}$  from  $x = -1$  to  $x = 1$ ?

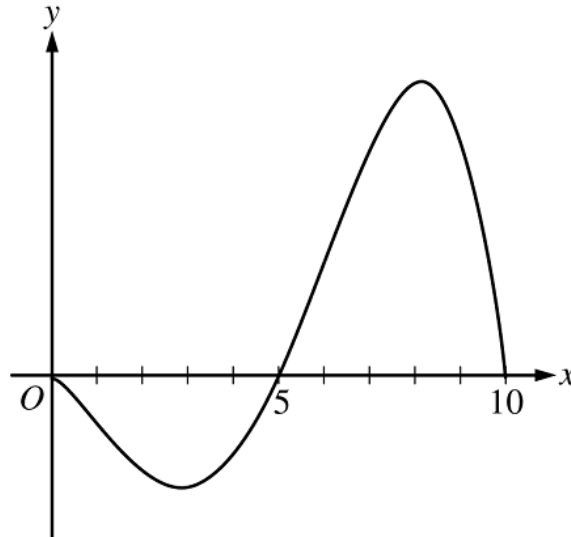
- (A)  $2\int_0^1 \sqrt{1 + \frac{9}{4}y} dy$     (B)  $\int_{-1}^1 \sqrt{1 + \frac{9}{4}y} dy$     (C)  $\int_0^1 \sqrt{1 + y^3} dy$     (D)  $\int_0^1 \sqrt{1 + y^6} dy$     (E)  $\int_0^1 \sqrt{1 + y^{9/4}} dy$

7. (AP BC 2002B-3) (Calculator Permitted) Let  $R$  be the region in the first quadrant bounded by the  $y$ -axis and the graphs of  $y = 4x - x^3 + 1$  and  $y = \frac{3}{4}x$ .



- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
- (c) Write an expression involving one or more integrals that gives the perimeter of  $R$ . Do not evaluate.

8. (AP BC 2011B-4) The graph of the differentiable function  $y = f(x)$  with domain  $0 \leq x \leq 10$  is shown in the figure at right. The area of the region enclosed between the graph of  $f$  and the  $x$ -axis for  $0 \leq x \leq 5$  is 10, and the area of the region enclosed between the graph of  $f$  and the  $x$ -axis for  $5 \leq x \leq 10$  is 27. The arc length for the portion of the graph of  $f$  between  $x = 0$  and  $x = 5$  is 11, and the arc length for the portion of the graph of  $f$  between  $x = 5$  and  $x = 10$  is 18. The function  $f$  has exactly two critical points that are located at  $x = 3$  and  $x = 8$ .

Graph of  $f$ 

- (a) Find the average value of  $f$  on the interval  $0 \leq x \leq 5$ .

- (b) Evaluate  $\int_0^{10} (3f(x) + 2) dx$ . Show the computations that lead to your answer.

- (c) Let  $g(x) = \int_5^x f(t) dt$ . On what intervals, if any, is the graph of  $g$  both concave up and decreasing? Explain your reasoning.

- (d) The function  $h$  is defined by  $h(x) = 2f\left(\frac{x}{2}\right)$ . The derivative of  $h$  is  $h'(x) = f'\left(\frac{x}{2}\right)$ . Find the arc length of the graph of  $y = h(x)$  from  $x = 0$  to  $x = 20$ .