Worksheet 6.4—Arc Length
Show all work. No calculator unless stated.

Multiple Choice

1. (‘88 BC) The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

(A) $\int_{0}^{2} \sqrt{1 + x^6} \, dx$  
(B) $\int_{0}^{2} \sqrt{1 + 3x^2} \, dx$  
(C) $\pi \int_{0}^{2} \sqrt{1 + 9x^4} \, dx$

(D) $2\pi \int_{0}^{2} \sqrt{1 + 9x^4} \, dx$  
(E) $\int_{0}^{2} \sqrt{1 + 9x^4} \, dx$

2. (‘03 BC) The length of a curve from $x = 1$ to $x = 4$ is given by $\int_{1}^{4} \sqrt{1 + 9x^4} \, dx$. If the curve contains the point $(1, 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$  
(B) $y = 5 + x^3$  
(C) $y = 6 + x^3$

(D) $y = 6 - x^3$  
(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$
3. (Calculator Permitted) Which of the following gives the best approximation of the length of the arc of 
\[ y = \cos(2x) \] from \( x = 0 \) to \( x = \frac{\pi}{4} \) ?

(A) 0.785       (B) 0.955       (C) 1.0       (D) 1.318       (E) 1.977

4. Which of the following gives the length of the graph of \( x = y^3 \) from \( y = -2 \) to \( y = 2 \) ?

(A) \( \int_{-2}^{2} \left(1 + y^6\right) dy \)       (B) \( \int_{-2}^{2} \sqrt{1 + y^6} dy \)       (C) \( \int_{-2}^{2} \sqrt{1 + 9y^4} dy \)       (D) \( \int_{-2}^{2} \sqrt{1 + x^2} dx \)       (E) \( \int_{-2}^{2} \sqrt{1 + x^4} dx \)
5. Find the length of the curve described by \( y = \frac{2}{3} x^{3/2} \) from \( x = 0 \) to \( x = 8 \).

(A) \( \frac{26}{3} \)  \hspace{1cm} (B) \( \frac{52}{3} \)  \hspace{1cm} (C) \( \frac{512\sqrt{2}}{15} \)  \hspace{1cm} (D) \( \frac{512\sqrt{2}}{15} + 8 \)  \hspace{1cm} (E) 96

6. Which of the following expressions should be used to find the length of the curve \( y = x^{2/3} \) from \( x = -1 \) to \( x = 1 \)?

(A) \( 2 \int_{0}^{1} \sqrt{1 + \frac{9}{4}} \, dy \)  \hspace{1cm} (B) \( \int_{-1}^{1} \sqrt{1 + \frac{9}{4}} \, dy \)  \hspace{1cm} (C) \( \int_{0}^{1} \sqrt{1 + y^3} \, dy \)  \hspace{1cm} (D) \( \int_{0}^{1} \sqrt{1 + y^6} \, dy \)  \hspace{1cm} (E) \( \int_{0}^{1} \sqrt{1 + y^{9/4}} \, dy \)
7. (AP BC 2002B-3) (Calculator Permitted) Let \( R \) be the region in the first quadrant bounded by the \( y \)-axis and the graphs of \( y = 4x - x^3 + 1 \) and \( y = \frac{3}{4}x \).

(a) Find the area of \( R \).

(b) Find the volume of the solid generated when \( R \) is revolved about the \( x \)-axis.

(c) Write an expression involving one or more integrals that gives the perimeter of \( R \). Do not evaluate.
8. (AP BC 2011B-4) The graph of the differentiable function \( y = f(x) \) with domain \( 0 \leq x \leq 10 \) is shown in the figure at right. The area of the region enclosed between the graph of \( f \) and the \( x \)-axis for \( 0 \leq x \leq 5 \) is 10, and the area of the region enclosed between the graph of \( f \) and the \( x \)-axis for \( 5 \leq x \leq 10 \) is 27. The arc length for the portion of the graph of \( f \) between \( x = 0 \) and \( x = 5 \) is 11, and the arc length for the portion of the graph of \( f \) between \( x = 5 \) and \( x = 10 \) is 18. The function \( f \) has exactly two critical points that are located at \( x = 3 \) and \( x = 8 \).

(a) Find the average value of \( f \) on the interval \( 0 \leq x \leq 5 \).

(b) Evaluate \( \int_0^5 (3f(x) + 2) \, dx \). Show the computations that lead to your answer.

(c) Let \( g(x) = \int_5^x f(t) \, dt \). On what intervals, if any, is the graph of \( g \) both concave up and decreasing? Explain your reasoning.

(d) The function \( h \) is defined by \( h(x) = 2f \left( \frac{x}{2} \right) \). The derivative of \( h \) is \( h'(x) = f' \left( \frac{x}{2} \right) \). Find the arc length of the graph of \( y = h(x) \) from \( x = 0 \) to \( x = 20 \).