Name____ Date________________________ Period______

Worksheet 6.4—Arc Length
Show all work. No calculator unless stated.

Multiple Choice

1. ('88 BC) The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

   (A) $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ (C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

   (D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$

2. ('03 BC) The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

   (A) $y = 3+3x^2$ (B) $y = 5+x^3$ (C) $y = 6+x^3$

   (D) $y = 6-x^3$ (E) $y = \frac{16}{5}+x+\frac{9}{5}x^5$

3. (Calculator Permitted) Which of the following gives the best approximation of the length of the arc of $y = \cos(2x)$ from $x = 0$ to $x = \frac{\pi}{4}$?

(A) 0.785 (B) 0.955 (C) 1.0 (D) 1.318 (E) 1.977

4. Which of the following gives the length of the graph of $x = y^3$ from $y = -2$ to $y = 2$?

(A) $\int_{-2}^{2} \left(1 + y^6\right) dy$ (B) $\int_{-2}^{2} \sqrt{1 + y^6}\, dy$ (C) $\int_{-2}^{2} \sqrt{1 + 9y^4}\, dy$ (D) $\int_{-2}^{2} \sqrt{1 + x^2}\, dx$ (E) $\int_{-2}^{2} \sqrt{1 + x^4}\, dx$

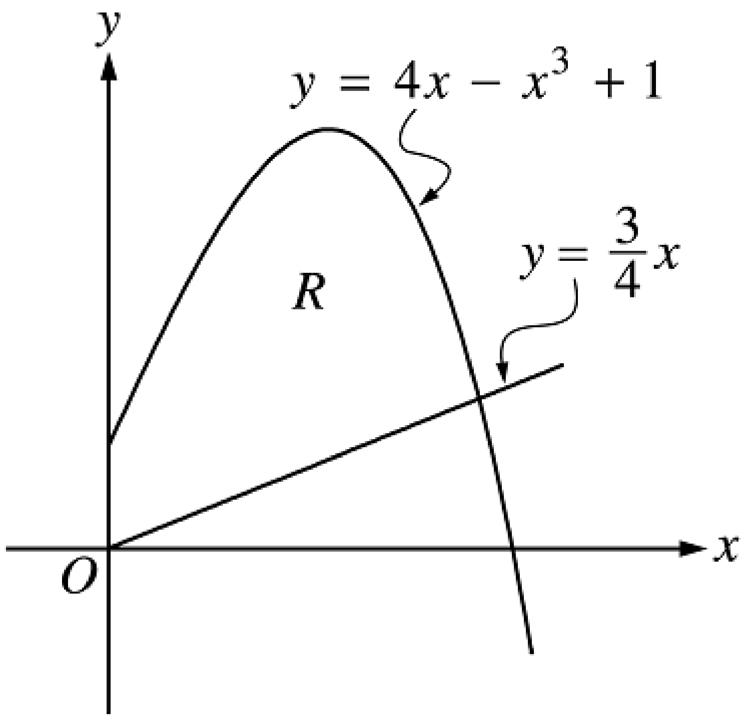
5. Find the length of the curve described by $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 8$.

(A) $\frac{26}{3}$ (B) $\frac{52}{3}$ (C) $\frac{512\sqrt{2}}{15}$ (D) $\frac{512\sqrt{2}}{15} + 8$ (E) 96

6. Which of the following expressions should be used to find the length of the curve $y = x^{2/3}$ from $x = -1$ to $x = 1$?

(A) $2\int_0^1 \sqrt{1+\frac{9}{4}y}\,dy$ (B) $\int_{-1}^1 \sqrt{1+\frac{9}{4}y}\,dy$ (C) $\int_0^1 \sqrt{1+y^3}\,dy$ (D) $\int_0^1 \sqrt{1+y^6}\,dy$ (E) $\int_0^1 \sqrt{1+y^{9/4}}\,dy$

7. (AP BC 2002B-3) (Calculator Permitted) Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

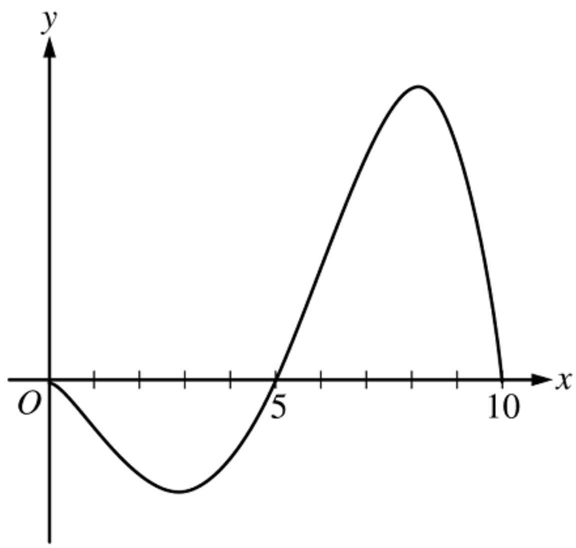


(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

8. (AP BC 2011B-4) The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure at right. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.



Graph of f

(a) Find the average value of f on the interval $0 \le x \le 5$.

(b) Evaluate $\int_0^{10} \left(3f(x) + 2\right) dx$. Show the computations that lead to your answer.

(c) Let $g(x) = \int_5^x f(t)\, dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.

(d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.