Name_____ Date_____ Period_____

Worksheet 6.5—L'Hôpital's Rule and Indeterminate Forms
Show all work. No calculator at all. Not one keystroke.

Multiple Choice

1. Which of the following gives the value of $\lim_{x\to 0}\frac{x}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) π (E) Does not exist

2. Which of the following gives the value of $\lim_{x\to 1}\frac{1-1/x}{1-1/x^2}$?

(A) Does not exist (B) 2 (C) 1 (D) 1/2 (E) 0

3. Which of the following gives the value of $\lim_{x\to\infty}\frac{\log_2 x}{\log_3 x}$?

(A) 1 (B) $\frac{\ln 3}{\ln 2}$ (C) $\frac{\ln 2}{\ln 3}$ (D) $\ln\left(\frac{3}{2}\right)$ (E) $\ln\left(\frac{2}{3}\right)$

4. Which of the following gives the value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$?

(A) 0 (B) 1 (C) e (D) e^2 (E) e^3

5. What is $\lim_{h\to 0}\frac{8\left(\frac{1}{2}+h\right)^8-8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) 1/2 (C) 1 (D) Does not exist (E) Cannot be determined from the information given

6. What is $\lim_{x\to 0}\frac{e^{2x}-1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) Does not exist

7. What is $\lim_{m\to 0}\frac{1}{m}\ln\left(\frac{2+m}{2}\right)$?

(A) e^2 (B) 1 (C) 1/2 (E) 0 (E) Does not exist

8. What is $\lim_{n\to\infty}\frac{4n^2}{10{,}000n+n^2}$?

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) Does not exist

9. $\lim_{x\to 0}(1+2x)^{\csc x} =$

(A) 0 (B) 1 (C) 2 (D) e (E) e^2

Free Response

Find the following limits. Show all work and use L'Hopital's Rule whenever possible. Express your answer in exact form. For example, if the value of the limit is π do not write 3.14

10. $\lim_{\theta\to 0}\dfrac{\arctan\theta}{2\theta}$

11. $\lim_{\theta\to \pi^+}\dfrac{1}{\sin(\theta-\pi)}$

12. $\lim_{x\to \pi^+}\dfrac{2x-2\pi}{\sin(x-\pi)}$

13. $\lim_{z\to \pi^+}\sin\left(\dfrac{1}{z-\pi}\right)$

14. $\lim_{\alpha\to 0}\alpha\cdot\cot(2\alpha)$

15. $\lim_{y\to\infty} y\cdot\ln\left(\dfrac{y+1}{y-1}\right)$

16. $\lim_{x\to 0^-} x^3\cdot e^{1/x}$

17. $\lim_{t\to\infty}\left(1+\dfrac{3}{t}\right)^t$

18. $\lim_{w\to\infty}\left(\ln w-\sqrt{w}\right)$

19. $\lim_{t\to\infty} \dfrac{\sin\left(\frac{1}{t}\right)}{\ln t}$

20. $\lim_{v\to\infty} \dfrac{v^2}{e^{-v}}$

21. $\lim_{x\to 0^+} \dfrac{x^2 \cdot \sin\left(\frac{1}{x}\right)}{\sin x}$

22. $\lim_{u\to 0^+} \left(2^u - 1\right)\sqrt{u}$

23. $\lim_{x\to\infty} \left(\ln|2x-4| - \ln|x+3|\right)$

24. $\lim_{\theta\to 0} \pi^2 \dfrac{\tan 2\theta}{\theta \cos 2\theta}$

25. (AP 2010-5) Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

26. (2016-BC4)

Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2,8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2,8)$? Justify you answer.

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$.

Find $\lim_{x \to -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right)$. Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$.

(BC) Use Euler's method starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

(AB) Use the linearization of $h(x)$ at $x = 0$ to approximate $h(1)$.