Name____ Date_______________________ Period_____

Worksheet 6.6—Improper Integrals
Show all work. No calculator unless explicitly stated.

Short Answer
1. Classify each of the integrals as proper or improper integrals. Give a clear reason for each.

(a) $\int_5^\infty \frac{dx}{(x-2)^2}$ (b) $\int_1^5 \frac{dx}{(x-2)^2}$ (c) $\int_2^5 \frac{dx}{(x-2)^2}$ (d) $\int_3^5 \frac{dx}{(x-2)^2}$

2. Answer the following.

(a) If $\int_a^\infty f(x)dx = K$ and $0 < g(x) \le f(x)$, what can we say about $\int_a^\infty g(x)dx$?

(b) If $\int_a^\infty f(x)dx = K$ and $0 < f(x) < g(x)$, what can we say about $\int_a^\infty g(x)dx$?

(c) If $\int_a^\infty f(x)dx$ diverges and $0 < f(x) \le g(x)$, what can we say about $\int_a^\infty g(x)dx$?

(d) If $\int_a^\infty f(x)dx$ diverges and $0 < g(x) < f(x)$, what can we say about $\int_a^\infty g(x)dx$?

3. If $\int_1^\infty \frac{1}{x^p}\,dx$ converges for $p > 1$, what can be said in general about improper integrals of the form $\int_a^\infty \frac{1}{x^p}\,dx$? For what values of a does the function diverge? Converge? To what ?

4. Determine if the improper integral converges or diverges by finding a function to compare it to. Justify by showing the inequality and discussing the convergence/divergence of the function to which you compare.

(a) $\int_2^\infty \frac{x^5}{x^6-1}dx$ (b) $\int_2^\infty \frac{x^3+1}{(x^4+4x+1)^2}dx$ (c) $\int_1^\infty \frac{dx}{(x+5)^5}$ (d) $\int_4^\infty \frac{3+\sin x}{x}dx$

Multiple Choice

5. $\int_0^\infty x^2 e^{-x^3}\,dx =$

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) Diverges

6. Which of the following gives the value of the integral $\int_1^\infty \frac{dx}{x^{1.01}}$?

(A) 1 (B) 10 (C) 100 (D) 1000 (E) Diverges

7. Which of the following gives the value of the integral $\int_0^1 \frac{dx}{x^{0.5}}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) Diverges

8. Which of the following gives the value of the integral $\int_0^1 \frac{dx}{x-1}$?

(A) -1 (B) $-1/2$ (C) 0 (D) 1 (E) Diverges

9. Which of the following gives the value of the area under the curve $y=\dfrac{1}{x^2+1}$ in the first quadrant?

(A) $\dfrac{\pi}{4}$ (B) 1 (C) $\dfrac{\pi}{2}$ (D) π (E) Diverges

10. Determine if $\displaystyle\int_0^2 f(x)dx$ is convergent or divergent when $f(x)=\begin{cases} x^{-1/2}, & x\le 1 \\ x, & 1<x\le 2 \end{cases}$, and if it is convergent, find its value.

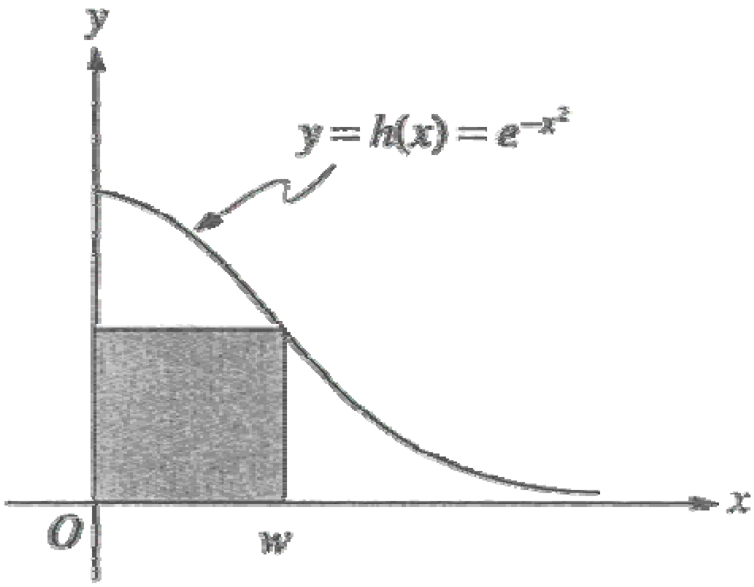
(A) 1/2 (B) 5/2 (C) 7/2 (D) 4 (E) Diverges

11. $\displaystyle\int_2^\infty \frac{x}{\sqrt[3]{x^2-2}}\,dx=$

(A) $\dfrac{3\cdot 2^{2/3}}{4}$ (B) $2^{2/3}$ (C) $-\dfrac{3\cdot 2^{2/3}}{4}$ (D) $-\dfrac{3\cdot 2^{2/3}}{2}$ (E) Diverges

Free Response

12. (AP 1996-1) Consider the graph of the function h given by $h(x) = e^{-x^2}$ for $0 \le x < \infty$.



(a) Let R be the unbounded region in the first quadrant below the graph of h. Find the volume of the solid generated when R is revolved about the y-axis.

(b) Let $A(w)$ be the area of the shaded rectangle shown in the figure. Show that $A(w)$ has its maximum value when w is the x-coordinate of the point of inflection of the graph of h.

13. (AP 2001-5) Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(a) Evaluate $\int_1^\infty -3xf(x)\,dx$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the intial condition $f(1) = 4$.

14. (AP 2010B-5) Let f and g be the functions defined by $f(x)=\frac{1}{x}$ and $g(x)=\frac{4x}{1+4x^2}$, for all $x>0$.

(a) Find the absolute maximum value of g on the open interval $(0,\infty)$ if the maximum exists. Find the absolute minimum value of g on the open interval $(0,\infty)$ if the minimum exists. Justify your answers.

(b) Find the area of the unbounded region in the first quadrant to the right of the vertical line $x=1$, below the graph of f, and above the graph of g.