Worksheet 7.1—Intro to Parametric & Vector Calculus
Show all work. No calculator unless explicitly stated.

Short Answer
1. If \( x = t^2 - 1 \) and \( y = e^t \), find \( \frac{dy}{dx} \).

2. If a particle moves in the \( xy \)-plane so that at any time \( t > 0 \), its position vector is \( \left( \ln(t^2 + 5t), 3t^2 \right) \), find its velocity vector at time \( t = 2 \).

3. A particle moves in the \( xy \)-plane so that at any time \( t \), its coordinates are given by \( x = t^5 - 1 \), \( y = 3t^4 - 2t^3 \). Find its acceleration vector at \( t = 1 \).
4. If a particle moves in the $xy$–plane so that at time $t$, its position vector is \( \left< \sin \left(3t - \frac{\pi}{2}\right), 3t^2 \right> \), find the velocity vector at time $t = \frac{\pi}{2}$.

5. A particle moves on the curve $y = \ln x$ so that its $x$-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1,0)$. Find the position of the particle at time $t = 1$.

6. A particle moves in the $xy$–plane in such a way that its velocity vector is $\left< 1 + t, t^3 \right>$. If the position vector at $t = 0$ is $\left< 5, 0 \right>$, find the position of the particle at $t = 2$. 
7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

8. The position of a particle moving in the $xy$–plane is given by the parametric equations

$$x = t^3 - \frac{3}{2}t^2 - 18t + 5 \quad \text{and} \quad y = t^3 - 6t^2 + 9t + 4.$$  For what value(s) of $t$ is the particle at rest?

9. A curve $C$ is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write an equation of the line tangent to the graph of $C$ at the point $(8, -4)$. 

10. (Calculator Permitted) A particle moves in the $xy$–plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$. Find the velocity vector at the time when the particle’s horizontal position is $x = 25$.

**Free Response:**

11. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at time $t = 5$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

(c) Find $\frac{dy}{dx}$ as a function of $x$. 
12. Point $P(x, y)$ moves in the $xy$-plane in such away that $\frac{dx}{dt} = \frac{1}{t + 1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$.

(a) Find the coordinates of $P$ in terms of $t$ when $t = 1$, $x = \ln 2$, and $y = 0$.

(b) Write an equation expressing $y$ in terms of $x$.

(c) Find the average rate of change of $y$ with respect to $x$ as $t$ varies from 0 to 4.

(d) Find the instantaneous rate of change of $y$ with respect to $x$ when $t = 1$. 
13. Consider the curve $C$ given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

(a) Find $\frac{dy}{dx}$ as a function of $t$.

(b) Find an equation of the tangent line at the point where $t = \frac{\pi}{4}$.

(c) (Calculator Permitted) The curve $C$ intersects the $y$-axis twice. Approximate the length of the curve between the two $y$-intercepts.
Multiple Choice:

14. A parametric curve is defined by \( x = \sin t \) and \( y = \csc t \) for \( 0 < t < \frac{\pi}{2} \). This curve is
   \( \text{(A) increasing & concave up} \quad \text{(B) increasing & concave down} \quad \text{(C) decreasing & concave up} \quad \text{(D) decreasing & concave down} \quad \text{(E) decreasing with a point of inflection} \)

15. The parametric curve defined by \( x = \ln t, \ y = t \) for \( t > 0 \) is identical to the graph of the function
   \( \text{(A) } y = \ln x \text{ for all real } x \quad \text{(B) } y = \ln x \text{ for } x > 0 \quad \text{(C) } y = e^x \text{ for all real } x \quad \text{(D) } y = e^x \text{ for } x > 0 \quad \text{(E) } y = \ln \left( e^x \right) \text{ for } x > 0 \)

16. The position of a particle in the \( xy \)-plane is given by \( x = t^2 + 1 \) and \( y = \ln (2t + 3) \) for all \( t \geq 0 \). The acceleration vector of the particle is
   \( \text{(A) } \left( 2t, \frac{2}{2t + 3} \right) \quad \text{(B) } \left( 2t, -\frac{4}{(2t + 3)^2} \right) \quad \text{(C) } \left( 2, -\frac{4}{(2t + 3)^2} \right) \quad \text{(D) } \left( 2, \frac{2}{(2t + 3)^2} \right) \quad \text{(E) } \left( 2, -\frac{4}{(2t + 3)^2} \right) \)