Worksheet 7.2—Parametric & Vector Accumulation
Show all work. No calculator except unless specifically stated.

Short Answer/Free Response

1. If \( x = e^{2t} \) and \( y = \sin(3t) \), find \( \frac{dy}{dx} \) in terms of \( t \).

2. Write an integral expression to represent the length of the path described by the parametric equations
\[
x = \cos^3 t \quad \text{and} \quad y = \sin^2 t \quad \text{for} \quad 0 \leq t \leq \frac{\pi}{2}
\]

3. For what value(s) of \( t \) does the curve given by the parametric equations \( x = t^3 - t^2 - 1 \) and \( y = t^4 + 2t^2 - 8t \) have a vertical tangent?
4. Find the equation of the tangent line to the curve given by the parametric equations \( x(t) = 3t^2 - 4t + 2 \) and \( y(t) = t^3 - 4t \) at the point on the curve where \( t = 1 \).

5. If \( x(t) = e^t + 1 \) and \( y = 2e^{2t} \) are the equations of the path of a particle moving in the \( xy \)-plane, write an equation for the path of the particle in terms of \( x \) and \( y \).

6. (Calculator) A particle moves in the \( xy \)-plane so that its position at any time \( t \) is given by \( x = \cos(5t) \) and \( y = t^3 \). What is the speed of the particle when \( t = 2 \)?
7. (Calculator) The position of a particle at time $t \geq 0$ is given by the parametric equations

$$x(t) = \frac{(t - 2)^3}{3} + 4 \quad \text{and} \quad y(t) = t^2 - 4t + 4.$$ 

(a) Find the magnitude of the velocity vector at $t = 1$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.

(c) When is the particle at rest? What is its position at that time?
8. (Calculator) An object moving along a curve in the \( xy \)-plane has position \((x(t), y(t))\) at time \( t \geq 0 \) with \( \frac{dx}{dt} = 1 + \tan(t^2) \) and \( \frac{dy}{dt} = 3e^{\sqrt{t}} \). Find the acceleration vector and the speed of the object when \( t = 5 \).

9. (Calculator) A particle moves in the \( xy \)-plane so that the position of the particle is given by \( x(t) = t + \cos t \) and \( y(t) = 3t + 2\sin t \), \( 0 \leq t \leq \pi \). Find the velocity vector when the particle’s vertical position is \( y = 5 \).
10. (Calculator) An object moving along a curve in the $xy$–plane has position $(x(t), y(t))$ at time $t$ with \[
\frac{dx}{dt} = 2\sin(t^3) \quad \text{and} \quad \frac{dy}{dt} = \cos(t^2) \text{ for } 0 \leq t \leq 4. \] At time $t = 1$, the object is at the position $(3, 4)$.

(a) Write an equation for the line tangent to the curve at $(3, 4)$.

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \leq t \leq 1$.

(d) Find the position of the object at time $t = 2$. 

Multiple Choice:

11. (Calculator) An object moving along a curve in the $xy$–plane has position $(x(t), y(t))$ with

$\frac{dx}{dt} = \cos(t^2)$ and $\frac{dy}{dt} = \sin(t^3)$. At time $t = 0$, the object is at position $(4, 7)$. Where is the particle when $t = 2$?

(A) $(-0.564, 0.989)$  (B) $(0.461, 0.452)$  (C) $(3.346, 7.989)$
(D) $(4.461, 7.452)$  (E) $(5.962, 8.962)$

12. (Calculator) The path of a particle moving in the plane is defined parametrically as a function of time $t$ by $x = \sin 2t$ and $y = \cos 5t$. What is the speed of the particle at $t = 2$?

(A) $1.130$  (B) $3.018$  (C) $\langle -1.307, 2.720 \rangle$  (D) $\langle 0.757, 0.839 \rangle$  (E) $\langle 1.307, 2.720 \rangle$
13. For what values of \( t \) does the curve given by the parametric equations \( x = t^3 - t^2 - 1 \) and 
\( y = t^4 + 2t^2 - 8t \) have a vertical tangent?

(A) 0 only       (B) 1 only       (C) 0 and 2/3 only       (D) 0, 2/3, and 1       (E) No value

14. The distance traveled by a particle from \( t = 0 \) to \( t = 4 \) whose position is given by the vector 
\( \vec{s}(t) = \langle t^2, t \rangle \) is given by

(A) \( \int_{0}^{4} \sqrt{4t + 1} \, dt \)       (B) \( 2 \int_{0}^{4} \sqrt{t^2 + 1} \, dt \)       (C) \( 4 \int_{0}^{4} \sqrt{2t^2 + 1} \, dt \)       (D) \( 4 \int_{0}^{4} \sqrt{4t^2 + 1} \, dt \)

(E) \( 2\pi \int_{0}^{4} \sqrt{4t^2 + 1} \, dt \)