① one petal of $r = 2\cos(3\theta)$

polar zeros: $2\cos 3\theta = 0$

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}$

using symmetry:

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/6} \left(2\cos(3\theta)\right)^2 d\theta\right]$$

* you can also integrate without symmetry from $\theta = \frac{\pi}{6}$ to $\theta = \frac{\pi}{2}$ (between consecutive polar zeros)

$$= 4\int_0^{\pi/6} \cos^2(3\theta)\, d\theta$$

*power-reducing ID

$$= 4\left(\tfrac{1}{2}\right)\int_0^{\pi/6} (1+\cos 6\theta)\, d\theta$$

$$= 2\left[\theta + \tfrac{1}{6}\sin(6\theta)\right]\Big|_0^{\pi/6}$$

$$= 2\left[\left(\tfrac{\pi}{6}+0\right) - (0+0)\right]$$

$$= \boxed{\frac{\pi}{3}}$$

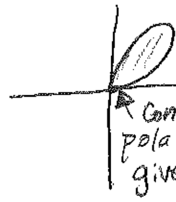
② one petal of $r = 4\sin(2\theta)$

polar zeros: $4\sin 2\theta = 0$

$\sin 2\theta = 0$

$2\theta = 0, \pi, \ldots$

$\theta = 0, \frac{\pi}{2}$



$$\text{Area} = \frac{1}{2}\int_0^{\pi/2} \left(4\sin(2\theta)\right)^2 d\theta$$

*square & pwr-reducing ID.

$$= 8\left(\tfrac{1}{2}\right)\int_0^{\frac{\pi}{2}} (1-\cos 4\theta)\, d\theta$$

$$= 4\left[\theta - \tfrac{1}{4}\sin(4\theta)\right]\Big|_0^{\pi/2}$$

$$= 4\left[\left(\tfrac{\pi}{2}-0\right) - (0-0)\right]$$

$$= \boxed{2\pi}$$

③ interior of $r = 2 + 2\cos\theta$

polar zeros: $2 + 2\cos\theta = 0$

$\cos\theta = -1$

$\theta = \pi$

using symmetry:

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi} (2+2\cos\theta)^2 d\theta\right]$$

$$= \int_0^{\pi} (4 + 8\cos\theta + 4\cos^2\theta)\, d\theta$$

$$= \int_0^{\pi} (4 + 8\cos\theta + 2(1+\cos 2\theta))\, d\theta$$

$$= \int_0^{\pi} (6 + 8\cos\theta + 2\cos 2\theta)\, d\theta$$

$$= 6\theta + 8\sin\theta + \sin 2\theta \Big|_0^{\pi}$$

$$= \left[(6\pi + 0 + 0) - (0+0+0)\right]$$

$$= \boxed{6\pi}$$

④ interior of $r = 2 - \sin\theta$

polar zeros: $2 - \sin\theta = 0$

$\sin\theta = 2 \rightarrow$ no zeros

$$\text{Area} = \frac{1}{2}\int_0^{2\pi} (2-\sin\theta)^2 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (4 - 4\sin\theta + \sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(4 - 4\sin\theta + \tfrac{1}{2}(1-\cos 2\theta)\right) d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(\tfrac{9}{2} - 4\sin\theta - \tfrac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\left[\tfrac{9}{2}\theta + 4\cos\theta - \tfrac{1}{4}\sin 2\theta\right]\Big|_0^{2\pi}$$

$$= \frac{1}{2}\left[(9\pi + 4 - 0) - (0 + 4 - 0)\right]$$

$$= \boxed{\frac{9\pi}{2}}$$

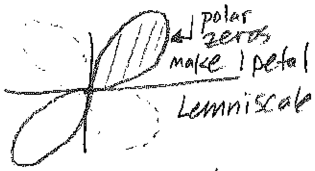
(5) interior of $r^2 = 4\sin(2\theta)$

polar zeros: $4\sin 2\theta = 0$

$\sin 2\theta = 0$

$2\theta = 0, \pi$

$\theta = 0, \frac{\pi}{2}$



using symmetry:

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/2}\underbrace{(4\sin(2\theta))}_{r^2 \text{ already}}\,d\theta\right]$$

$$= 4\int_0^{\pi/2}\sin 2\theta\,d\theta$$

$$= -2\cos 2\theta\Big|_0^{\pi/2}$$

$$= -2[\cos\pi - \cos 0]$$

$$= -2[-1-1]$$

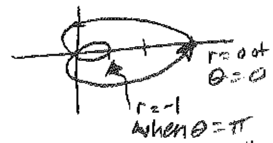
$$= \boxed{4}$$

(6) inner loop $r = 1 + 2\cos\theta$

polar zeros: $1 + 2\cos\theta = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ (define inner loop)



$$\text{Area} = \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(1+2\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(1 + 4\cos\theta + 4\cos^2\theta)\,d\theta = \frac{1}{2}\int_{2\pi/3}^{4\pi/3}\left(1 + 4\cos\theta + 4\left(\tfrac{1}{2}\right)(1+\cos 2\theta)\right)d\theta$$

$$= \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3 + 4\cos\theta + 2\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\left[3\theta + 4\sin\theta + \sin 2\theta\right]\Big|_{2\pi/3}^{4\pi/3}$$

$$= \frac{1}{2}\left[\left(4\pi + 4\sin\tfrac{4\pi}{3} + \sin\tfrac{8\pi}{3}\right) - \left(2\pi + 4\sin\tfrac{2\pi}{3} + \sin\tfrac{4\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[4\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2} - 2\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2}\right]$$

$$= \frac{1}{2}\left[2\pi - 4\sqrt{3} + \sqrt{3}\right] = \frac{1}{2}\left[2\pi - 3\sqrt{3}\right]$$

$$= \boxed{\pi - \frac{3\sqrt{3}}{2}}$$

(7) between loops of $r = 1 + 2\cos\theta$

* same curve as in #6.

using symmetry:

$$\text{Area} = \underbrace{2\left[\frac{1}{2}\int_0^{\frac{2\pi}{3}}(1+2\cos\theta)^2\,d\theta\right]}_{\text{outer}} - \underbrace{\left(\pi - \frac{3\sqrt{3}}{2}\right)}_{\text{inner (from \#6)}}$$

$$= \int_0^{\frac{2\pi}{3}}(1 + 4\cos\theta + 4\cos^2\theta)\,d\theta - \pi + \frac{3\sqrt{3}}{2}$$

$$= \int_0^{\frac{2\pi}{3}}(3 + 4\cos\theta + 2\cos 2\theta)\,d\theta - \pi + \frac{3\sqrt{3}}{2}$$

$$= \left[3\theta + 4\sin\theta + \sin 2\theta\right]\Big|_0^{2\pi/3} - \pi + \frac{3\sqrt{3}}{2}$$

$$= \left[\left(2\pi + 4\sin\tfrac{2\pi}{3} + \sin\tfrac{4\pi}{3}\right) - (0+0+0)\right] - \pi + \frac{3\sqrt{3}}{2}$$

$$= 2\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2} - \pi + \frac{3\sqrt{3}}{2}$$

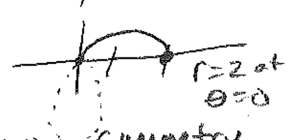
$$= \boxed{\pi + 3\sqrt{3}}$$

(8) one loop of $r^2 = 4\cos(2\theta)$

polar zeros: $\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$



using symmetry

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\frac{\pi}{4}}\underbrace{(4\cos(2\theta))}_{\text{already squared}}\,d\theta\right]$$

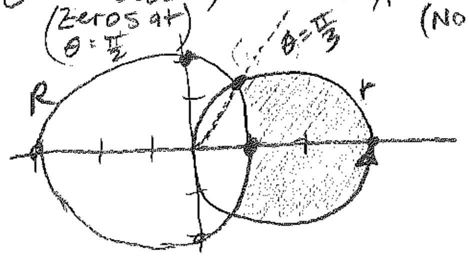
$$= 2\sin(2\theta)\Big|_0^{\pi/4}$$

$$= 2\sin\frac{\pi}{2} - 2\sin 0$$

$$= \boxed{2}$$

(9) inside $r = 3\cos\theta$, outside $R = 2 - \cos\theta$

(zeros at $\theta = \frac{\pi}{2}$) (No zeros)



point of intersection:

$3\cos\theta = 2 - \cos\theta$

$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}}$

using symmetry:

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\frac{\pi}{3}}(3\cos\theta)^2 d\theta - \frac{1}{2}\int_0^{\pi/3}(2-\cos\theta)^2 d\theta\right]$$

$$= \int_0^{\pi/3}\left[9\cos^2\theta - (4 - 4\cos\theta + \cos^2\theta)\right]d\theta$$

$$= \int_0^{\pi/3}\left[8\cos^2\theta + 4\cos\theta - 4\right]d\theta$$

$$= \int_0^{\pi/3}\left[4(1+\cos 2\theta) + 4\cos\theta - 4\right]d\theta$$

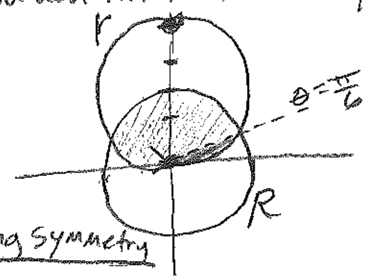
$$= \int_0^{\frac{\pi}{3}}\left[4\cos(2\theta) + 4\cos\theta\right]d\theta$$

$$= 2\sin(2\theta) + 4\sin\theta \Big|_0^{\pi/3}$$

$$= \left(2\sin\left(\frac{2\pi}{3}\right) + 4\sin\left(\frac{\pi}{3}\right)\right) - (0)$$

$$= \sqrt{3} + 2\sqrt{3} = \boxed{3\sqrt{3}}$$

(10) Common interior of $r = 4\sin\theta$, $R = 2$



point of intersect

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

using symmetry

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/6}(4\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2}(2)^2 d\theta\right]$$

$$= \int_0^{\pi/6} 16\sin^2\theta\, d\theta + \int_{\pi/6}^{\pi/2} 4\, d\theta$$

$$= 8\int_0^{\pi/6}(1-\cos 2\theta)\,d\theta + \int_{\pi/6}^{\pi/2} 4\, d\theta$$

$$= 8\left[\theta - \frac{1}{2}\sin 2\theta\right]\Big|_0^{\pi/6} + 4\theta\Big|_{\pi/6}^{\pi/2}$$

$$= 8\left[\left(\frac{\pi}{6} - \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right) - (0-0)\right] + 4\left[\frac{\pi}{2} - \frac{\pi}{6}\right]$$

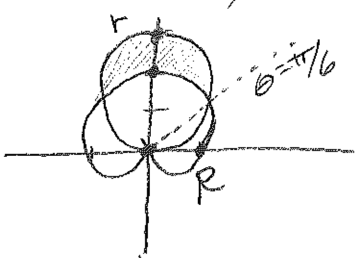
$$= 8\left[\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right] + 4\left[\frac{\pi}{3}\right]$$

$$= \frac{4\pi}{3} - 2\sqrt{3} + \frac{4\pi}{3}$$

$$= \boxed{\frac{8\pi}{3} - 2\sqrt{3}}$$

(11) inside $r = 3\sin\theta$, outside $R = 1+\sin\theta$

zero at $\theta = \frac{3\pi}{2}$



point of intersect

$3\sin\theta = 1+\sin\theta$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

using symmetry:

$$\text{Area} = 2\left[\frac{1}{2}\int_{\pi/6}^{\pi/2}(3\sin\theta)^2\,d\theta - \frac{1}{2}\int_{\pi/6}^{\pi/2}(1+\sin\theta)^2\,d\theta\right]$$

$$= 9\int_{\pi/6}^{\pi/2}\sin^2\theta\,d\theta - \int_{\pi/6}^{\pi/2}(1+2\sin\theta+\sin^2\theta)\,d\theta$$

$$= \int_{\pi/6}^{\pi/2}\left[\frac{9}{2}(1-\cos 2\theta) - 1 - 2\sin\theta - \frac{1}{2}(1-\cos 2\theta)\right]d\theta$$

$$= \int_{\pi/6}^{\pi/2}\left[3 - 4\cos 2\theta - 2\sin\theta\right]d\theta$$

$$= 3\theta - 2\sin 2\theta + 2\cos\theta\Big|_{\pi/6}^{\pi/2}$$

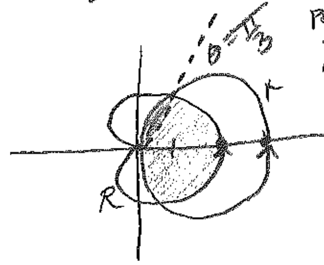
$$= \left[\frac{3\pi}{2} - 0 + 0\right] - \left[\frac{\pi}{2} - \sqrt{3} + \sqrt{3}\right]$$

$$= \frac{3\pi}{2} - \frac{\pi}{2}$$

$$= \boxed{\pi}$$

(12) common interior of $r = 3\cos\theta$, $R = 1+\cos\theta$

Polar zero at $x = \frac{\pi}{2}$ zero: $\cos\theta = -1$, $\theta = \pi$



intersection:

$3\cos\theta = 1+\cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

(using symmetry)

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/3}(1+\cos\theta)^2\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2}(3\cos\theta)^2\,d\theta\right]$$

$$= \int_0^{\pi/3}\left(1 + 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right)d\theta + \int_{\pi/3}^{\pi/2}\frac{9}{2}(1+\cos 2\theta)\,d\theta$$

$$= \int_0^{\pi/3}\left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right)d\theta + \frac{9}{2}\int_{\pi/3}^{\pi/2}(1+\cos 2\theta)\,d\theta$$

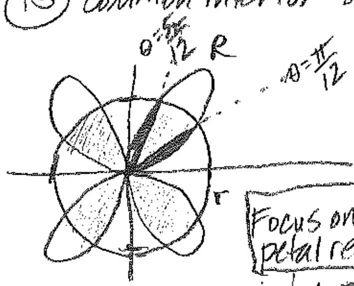
$$= \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]\Big|_0^{\pi/3} + \frac{9}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]\Big|_{\pi/3}^{\pi/2}$$

$$= \left[\left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right) - (0)\right] + \frac{9}{2}\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)\right]$$

$$= \frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} + \frac{9\pi}{4} - \frac{3\pi}{2} - \frac{9\sqrt{3}}{8}$$

$$= \boxed{\frac{5\pi}{4}}$$

(13) common interior of $r = 4\sin(2\theta)$ & $R = 2$



Focus on 1 petal region

zeros:
$\sin 2\theta = 0$
$2\theta = 0, \pi$
$\theta = 0, \frac{\pi}{2}$
(one petal)

intersection
$4\sin 2\theta = 2$
$\sin 2\theta = \frac{1}{2}$
$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$

1: Area contained by 1 Rose Curve
(using symmetry)

$$\text{Area}_1 = 2\left[\frac{1}{2}\int_0^{\pi/12} (4\sin 2\theta)^2\, d\theta\right]$$
$$= 16\int_0^{\pi/12} \sin^2 2\theta\, d\theta$$
$$= 8\int_0^{\pi/12} (1-\cos 4\theta)\, d\theta$$
$$= 8\left[\theta - \frac{1}{4}\sin 4\theta\right]\Big|_0^{\pi/12}$$
$$= 8\left[\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) - (0)\right]$$
$$= \boxed{\frac{2\pi}{3} - \sqrt{3}}$$

2: Area of 1 sector Contained by circle
(using symmetry)

$$\text{Area}_2 = \int_{\pi/12}^{5\pi/12} (2)^2\, d\theta$$
$$= 4\theta\Big|_{\pi/12}^{5\pi/12}$$
$$= 4\left[\frac{5\pi}{12} - \frac{\pi}{12}\right] = \boxed{\frac{4\pi}{3}}$$

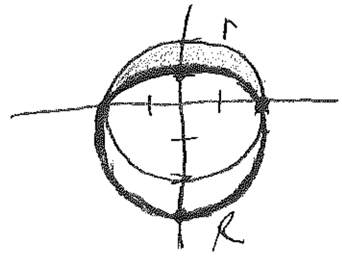
Total common Area:

$$\text{Area} = 4\left[\left(\frac{2\pi}{3} - \sqrt{3}\right) + \left(\frac{4\pi}{3}\right)\right]$$

4 regions

$$= 4\left[2\pi - \sqrt{3}\right]$$
$$= \boxed{8\pi - 4\sqrt{3}}$$

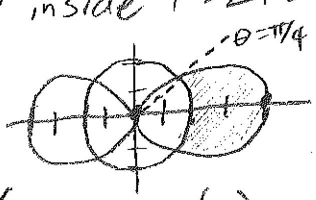
(14) inside $r = 2$ & outside $R = 2 - \sin\theta$



(using symmetry)

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/2} (2)^2\, d\theta - \frac{1}{2}\int_0^{\pi/2} (2-\sin\theta)^2\, d\theta\right]$$
$$= \int_0^{\pi/2} \left[4 - (4 - 4\sin\theta + \sin^2\theta)\right] d\theta$$
$$= \int_0^{\pi/2} \left[4\sin\theta - \frac{1}{2}(1-\cos 2\theta)\right] d\theta$$
$$= \int_0^{\pi/2} \left[4\sin\theta - \frac{1}{2} + \frac{1}{2}\cos 2\theta\right] d\theta$$
$$= -4\cos\theta - \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\Big|_0^{\pi/2}$$
$$= \left(0 - \frac{\pi}{4} + 0\right) - (-4)$$
$$= \boxed{4 - \frac{\pi}{4}}$$

(15) inside $r = 2 + 2\cos(2\theta)$ & outside $R = 2$



zeros of r:
$\cos 2\theta = -1$
$2\theta = \pi$
$\theta = \frac{\pi}{2}$

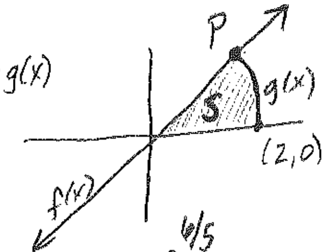
intersect:
$2 + 2\cos 2\theta = 2$
$\cos 2\theta = 0$
$\theta = \frac{\pi}{4}$

(using symmetry)

$$\text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/4} (2+2\cos(2\theta))^2\, d\theta - \frac{1}{2}\int_0^{\pi/4} (2)^2\, d\theta\right]$$
$$= \int_0^{\pi/4} \left((2+2\cos(2\theta))^2 - 4\right) d\theta$$
$$= \boxed{5.5707} \text{ (from calculator)}$$

(Dang, this is a LONG worksheet, but totally worth it!)

(16) $y = \frac{2}{3}x = f(x)$, $y = \sqrt{1 - \frac{x^2}{4}} = g(x)$



* x-intercept of $g(x)$

$1 = \frac{x^2}{4}$, $x^2 = 4$, $\boxed{x = 2}$

$\sqrt{1-\frac{x^2}{4}} = \sqrt{\frac{4-x^2}{4}} = \frac{1}{2}\sqrt{4-x^2}$

(a) point of intersection

$\frac{2}{3}x = \sqrt{1 - \frac{x^2}{4}}$

$\frac{4}{9}x^2 = 1 - \frac{1}{4}x^2$

$\left(\frac{4}{9} + \frac{1}{4}\right)x^2 = 1$

$x^2 = \frac{36}{25}$

$x = \cancel{-\frac{6}{5}}$ or $\boxed{\frac{6}{5}}$

$y = \frac{2}{3}\left(\frac{6}{5}\right) = \boxed{\frac{4}{5}}$

$\boxed{\text{So Point P is } \left(\frac{6}{5}, \frac{4}{5}\right)}$

(b) $\text{Area} = \int_0^{6/5}\left(\frac{2}{3}x - 0\right)dx + \int_{6/5}^{2}\left(\sqrt{1 - \frac{x^2}{4}} - 0\right)dx$

$= \int_0^{6/5} \frac{2}{3}x\,dx + \int_{6/5}^{2} \frac{1}{2}\sqrt{4-x^2}\,dx$ — need calculator to integrate

$= \boxed{0.9272956436}$

(c) Polar eq. for curve C

$y = \sqrt{1 - \frac{1}{4}x^2}$, $y = r\sin\theta$, $x = r\cos\theta$, $x^2 + y^2 = r^2$

$y^2 = 1 - \frac{1}{4}x^2$

$x^2 + 4y^2 = 4$

$(r\cos\theta)^2 + 4(r\sin\theta)^2 = 4$

$r^2\cos^2\theta + 4r^2\sin^2\theta = 4$

$\boxed{r^2 = \frac{4}{\cos^2\theta + 4\sin^2\theta}}$

or $\boxed{r = \frac{2}{\sqrt{\cos^2\theta + 4\sin^2\theta}}}$

(d) $y = \frac{2}{3}x$

$= r\sin\theta = \frac{2}{3}(r\cos\theta)$

$\tan\theta = \frac{2}{3}$

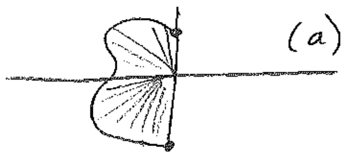
$\boxed{\theta = 0.588 \text{ rads} = A}$

$\text{Area} = \frac{1}{2}\int_0^A \left(\frac{4}{\cos^2\theta + 4\sin^2\theta}\right)d\theta$

$= \boxed{0.927295218}$

* note answers from (b) & (d) are equivalent to 5 decimals only because of the different ways the calculator numerically computes the integrals.

(17) $r = \theta + \cos(3\theta)$, $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$



(a) $\text{Area} = \frac{1}{2}\int_{\pi/2}^{3\pi/2} (\theta + \cos 3\theta)^2 \, d\theta = \boxed{1.610}$

(b) $y = -1$

$y = r\sin\theta = (\theta + \cos 3\theta)\sin\theta = -1$

* from calculator

$\boxed{\theta = 3.484 \text{ or } \theta = 3.485}$

(c) $\frac{dr}{d\theta} = 1 - 3\sin(3\theta) = 0$

* from calculator

$\theta = 2.207 = A,\ 3.028 = B,\ 4.302 = C$

$\frac{dr}{d\theta} > 0$ for $\theta \in \left[\frac{\pi}{2}, 2.207\right) \cup (3.028, 4.302)$. On these intervals the radius is increasing with respect to θ, which means the curve is moving away from the pole/origin.

(d) the distance from the origin $= r$, so we want to maximize r on $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$\frac{dr}{d\theta} = 1 - 3\sin(3\theta) = 0$

at $\theta = A, B, C$ (from part (c)). These are the critical values.

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ are the endpts.

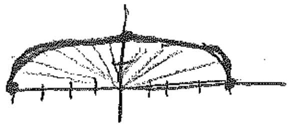
* using the EVT and the closed interval argument.

from calculator

$$\underbrace{\begin{cases} r(A) = 3.150 \\ r(B) = 2.085 \\ r(C) = 5.244 \\ r\left(\frac{\pi}{2}\right) = 1.570 \\ r\left(\frac{3\pi}{2}\right) = 4.712 \end{cases}}_{\text{Justification}}$$

So the curve is the greatest distance from the origin when $\theta = 4.302$ radians. At this time it is 5.244 units from the origin.

(18) $r = \dfrac{4}{1+\sin\theta}$, $0 \le \theta \le \pi$



(a) $\text{Area} = \frac{1}{2}\int_0^{\pi}\left(\frac{4}{1+\sin\theta}\right)^2 d\theta$

$= \boxed{10.667 = \frac{32}{3}}$

(b) $r = \dfrac{4}{1+\sin\theta}$

$r = \dfrac{4}{1+\frac{y}{r}}\left(\frac{r}{r}\right)$

$\dfrac{\cancel{r}}{1} = \dfrac{4\cancel{r}}{r+y}$

$r + y = 4$, $\sqrt{x^2+y^2} = 4 - y$

$x^2 + \cancel{y^2} = 16 - 8y + \cancel{y^2}$

$\boxed{8y = 16 - x^2}$

(c) $y = \frac{1}{8}[16 - x^2]$

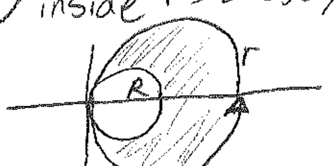
$y = 2 - \frac{1}{8}x^2$, x-ints at $x = -4, 4$

$\boxed{\text{Area} = \int_{-4}^{4}\left(2 - \tfrac{1}{8}x^2\right)dx}$

or using symmetry

$\text{Area} = 2\int_0^4\left(2 - \frac{1}{8}x^2\right)dx$

(19) inside $r = 2\cos\theta$, outside $R = \cos\theta$



1 revolution for $\theta \in [0, \pi]$

$\text{Area} = \frac{1}{2}\int_0^{\pi}[2\cos\theta]^2 d\theta - \frac{1}{2}\int_0^{\pi}[\cos\theta]^2 d\theta$

$= \frac{1}{2}\int_0^{\pi}[4\cos^2\theta - \cos^2\theta]\,d\theta$

$= \frac{1}{2}\int_0^{\pi}3\cos^2\theta\, d\theta$

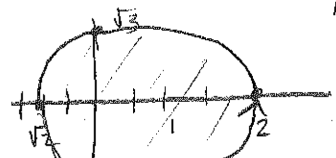
$= \frac{3}{2}\int_0^{\pi}\cos^2\theta\, d\theta$ answer not there

So... using symmetry

$= 2\left[\frac{3}{2}\int_0^{\pi/2}\cos^2\theta\, d\theta\right]$

$= \boxed{3\int_0^{\pi/2}\cos^2\theta\, d\theta}$ $\boxed{A}$

(20) $r = \sqrt{3+\cos\theta}$



trace out once from 0 to 2π

$\text{Area} = \frac{1}{2}\left[\int_0^{2\pi}\left(\sqrt{3+\cos\theta}\right)^2 d\theta\right]$

$= \frac{1}{2}\int_0^{2\pi}(3+\cos\theta)\,d\theta$

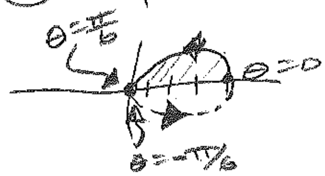
*using symmetry

$= 2\left[\frac{1}{2}\int_0^{\pi}(3+\cos\theta)\,d\theta\right]$

$= \boxed{\int_0^{\pi}(3+\cos\theta)\,d\theta}$

$\boxed{D}$

(21) one petal of $r = 4\cos(3\theta)$



polar zeros:

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}$

From answer choices:

$A = \frac{1}{2}\left[\int_{-\frac{\pi}{6}}^{\pi/6}(4\cos 3\theta)^2 d\theta\right]$

$= \boxed{8\int_{-\pi/6}^{\pi/6}\cos^2 3\theta\, d\theta}$ $\boxed{E}$

Finished already!