

Name _____ Date _____ Period _____

Worksheet 8.2—Polar AreaShow all work. **Calculator permitted** except unless specifically stated.**Short Answer:** Sketch a graph, shade the region, and find the area.

1. one petal of $r = 2 \cos(3\theta)$

2. one petal of $r = 4 \sin(2\theta)$

3. interior of $r = 2 + 2 \cos \theta$
(no calculator)

4. interior of $r = 2 - \sin \theta$
(no calculator)

5. interior of $r^2 = 4\sin(2\theta)$

6. inner loop of $r = 1 + 2\cos\theta$

7. between the loops of $r = 1 + 2\cos\theta$

8. one loop of $r^2 = 4\cos(2\theta)$

9. inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$

10. common interior of $r = 4\sin\theta$ and $r = 2$

11. inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$

12. common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$

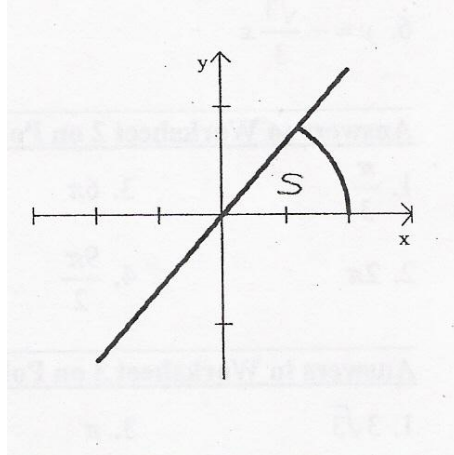
13. common interior of $r = 4\sin(2\theta)$ and $r = 2$

14. inside $r = 2$ and outside $r = 2 - \sin \theta$

15. inside $r = 2 + 2\cos(2\theta)$ and outside $r = 2$

Free Response

16. The figure shows the graphs of the line $y = \frac{2}{3}x$ and the curve C given by $y = \sqrt{1 - \frac{x^2}{4}}$. Let S be the region in the first quadrant bounded by the two graphs and the x -axis. The line and the curve intersect at point P .



- (a) Find the coordinates of P .
- (b) Set up and evaluate an integral expression with respect to x that gives the area of S .
- (b) Find a polar equation to represent curve C .
- (d) Use the polar equation found in (c) to set up and evaluate an integral expression with respect to the polar angle θ that gives the area of S .

17. A curve is drawn in the xy -plane and is described by the equation in polar coordinates $r = \theta + \cos(3\theta)$ for $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$, where r is measured in meters and θ is measured in radians.
- (a) Find the area bounded by the curve and the y -axis.

(b) Find the angle θ that corresponds to the point on the curve with y -coordinate -1 .

(c) For what values of θ , $\pi \leq \theta \leq \frac{3\pi}{2}$ is $\frac{dr}{d\theta}$ positive? What does this say about r ?

(d) Find the value of θ on the interval $\pi \leq \theta \leq \frac{3\pi}{2}$ that corresponds to the point on the curve with the greatest distance from the origin. What is this greatest distance? Justify your answer.

18. A region R in the xy -plane is bounded below by the x -axis and above by the polar curve defined by

$$r = \frac{4}{1 + \sin \theta} \text{ for } 0 \leq \theta \leq \pi.$$

(a) Find the area of R by evaluating an integral in polar coordinates.

(b) The curve resembles an arch of the parabola $8y = 16 - x^2$. Convert the polar equation to rectangular coordinates, and prove that the curves are the same.

(c) Set up an integral in rectangular coordinates that gives the area of R .

Multiple Choice

19. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (B) $3\int_0^{\pi} \cos^2\theta d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta d\theta$ (E) $3\int_0^{\pi} \cos\theta d\theta$

20. The area of the region enclosed by the polar graph of $r = \sqrt{3 + \cos\theta}$ is given by which integral?

(A) $\int_0^{2\pi} \sqrt{3 + \cos\theta} d\theta$ (B) $\int_0^{\pi} \sqrt{3 + \cos\theta} d\theta$ (C) $2\int_0^{\pi/2} (3 + \cos\theta) d\theta$
 (D) $\int_0^{\pi} (3 + \cos\theta) d\theta$ (E) $\int_0^{\pi/2} \sqrt{3 + \cos\theta} d\theta$

21. The area enclosed by one petal of the 3-petaled rose curve $r = 4\cos(3\theta)$ is given by which integral?

(A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta) d\theta$ (B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta) d\theta$ (C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta) d\theta$
 (D) $16\int_{-\pi/6}^{\pi/6} \cos(3\theta) d\theta$ (E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta) d\theta$