Worksheet 9.3—Power Series: Taylor and Maclaurin Series
Show all work. No calculator except unless specifically stated.

On problems 1-3, find a Taylor series for \( f(x) \) centered at the given value of \( a \). Give the first four nonzero terms and the general term for each series.

1. \( f(x) = e^{2x}, \ a = 3 \)
2. \( f(x) = \frac{1}{x}, \ a = 1 \)
3. \( f(x) = \ln x, \ a = 1 \)

On problems 4-5, find a Taylor series for \( f(x) \) centered at the given value of \( a \). Give the first four nonzero terms.

4. \( f(x) = \sin x, \ a = \frac{\pi}{6} \)
5. \( f(x) = \cos x, \ a = -\frac{\pi}{4} \)
On problems 6-10, find a Maclaurin series for \( f(x) \). Give the first four nonzero terms and the general term for each series. Hint: Don’t reinvent the wheel (or the series), rather, modify an existing power series.

6. \( f(x) = e^{\frac{x}{2}} \)  
7. \( f(x) = \sin(x^2) \)  
8. \( f(x) = \frac{\cos(3x)}{x} \)  
9. \( f(x) = x^2 e^{-x} \)

10. \( f(x) = \sin^2 x \) (ADDITIONAL HINT: use the power-reducing identity)
11. (Calculator Permitted) Use your answer for problem 7 to approximate \( \int_0^1 \sin(x^2) \, dx \) correct to three decimal places.

12. (a) Find the first four nonzero terms in the Taylor series expansion about (centered at) \( x = 0 \) for \( f(x) = \sqrt{1 + x} \).

(b) Use the results found in part (a) to find the first four nonzero terms in the Taylor series expansion about \( x = 0 \) for \( g(x) = \sqrt{1 + x^3} \).

(c) Find the first four nonzero terms in the Taylor series expansion about \( x = 0 \) for the function \( h \) such that \( h'(x) = \sqrt{1 + x^3} \) and \( h(0) = 4 \).
13. Let $f$ be the function defined by $f(x) = \frac{1}{x-1}$.

(a) Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x = 2$.

(b) Use the result from part (a) to find the first four terms and the general term of the series expansion about $x = 2$ for $\ln|x-1|$. 