

Name _____ Date _____ Period _____

Worksheet 9.5—Lagrange Error Bound

Show all work. Calculator permitted except unless specifically stated.

Free Response & Short Answer

1. (a) Find the fourth-degree Taylor polynomial for $\cos x$ about $x = 0$. Then use your polynomial to approximate the value of $\cos 0.8$, and use Taylor's Theorem to determine the accuracy of the approximation. Give three decimal places.

(b) Find the interval $[a, b]$ such that $a \leq \cos 0.8 \leq b$.

(c) Could $\cos 0.8$ equal 0.695? Show why or why not.

2. (a) Write a fourth-degree Maclaurin polynomial for $f(x) = e^x$. Then use your polynomial to approximate e^{-1} , and find a Lagrange error bound for the maximum error when $|x| \leq 1$. Give three decimal places.

(b) Find an interval $[a, b]$ such that $a \leq e^{-1} \leq b$.

3. Let f be a function that has derivatives of all orders for all real numbers x . Assume that $f(5) = 6$, $f'(5) = 8$, $f''(5) = 30$, $f'''(5) = 48$, and $|f^{(4)}(x)| \leq 75$ for all x in the interval $[5, 5.2]$.
- (a) Find the third-degree Taylor polynomial about $x = 5$ for $f(x)$.
- (b) Use your answer to part (a) to estimate the value of $f(5.2)$. What is the maximum possible error in making this estimate? Give three decimal places.
- (c) Find an interval $[a, b]$ such that $a \leq f(5.2) \leq b$. Give three decimal places.
- (d) Could $f(5.2)$ equal 8.254? Show why or why not.

Review (Problems 4 - 7):

4. Find the first four nonzero terms of the power series for $f(x) = \sin x$ centered at $x = \frac{3\pi}{4}$.

5. Find the first four nonzero terms and the general term for the Maclaurin series for

(a) $f(x) = x \cos(x^3)$

(b) $g(x) = \frac{1}{1+x^2}$

6. Find the radius and interval of convergence for

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{3^n n^2}$

(b) $\sum_{n=0}^{\infty} (2n)!(x-5)^n$

7. Use the Maclaurin series for $\cos x$ to find $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$.

8. The Taylor series about $x = 3$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 3$ is given by

$$f^{(n)}(3) = \frac{(-1)^n n!}{5^n (n+3)} \text{ and } f(3) = \frac{1}{3}$$

(a) Write the fourth-degree Taylor polynomial for f about $x = 3$.

(b) Find the radius of convergence of the Taylor series for f about $x = 3$.

(c) Show that the third-degree Taylor polynomial approximates $f(4)$ with an error less than $\frac{1}{4000}$.

9. Let f be a function that has derivatives of all orders on the interval $(-1,1)$. Assume $f(0) = 1$,

$$f'(0) = \frac{1}{2}, \quad f''(0) = -\frac{1}{4}, \quad f'''(0) = \frac{3}{8}, \quad \text{and} \quad |f^{(4)}(x)| \leq 6 \quad \text{for all } x \text{ in the interval } (-1,1).$$

(a) Find the third-degree Taylor polynomial about $x = 0$ for the function f .

(b) Use your answer to part (a) to estimate the value of $f(0.5)$.

(d) What is the maximum possible error for the approximation made in part (b)?

10. Let f be the function defined by $f(x) = \sqrt{x}$.

(a) Find the second-degree Taylor polynomial about $x = 4$ for the function f .

(b) Use your answer to part (a) to estimate the value of $f(4.2)$.

(c) Find a bound on the error for the approximation in part (b).

11. Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{2^n}$ for all x for which the series converges.

(a) Find the interval of convergence of this series.

(b) Use the first three terms of this series to approximate $f\left(-\frac{1}{2}\right)$.

(c) Estimate the error involved in the approximation in part (b). Show your reasoning.

12. Let f be the function given by $f(x) = \cos\left(3x + \frac{\pi}{6}\right)$ and let $P(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$.
- (a) Find $P(x)$.

- (b) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{6}\right) - P\left(\frac{1}{6}\right)\right| < \frac{1}{3000}$.

13. (Review) Use series to find an estimate for $I = \int_0^1 e^{-x^2} dx$ that is within 0.001 of the actual value.

Justify.

14. The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 5$ is given by

$$f^{(n)}(5) = \frac{(-1)^n n!}{2^n (n+2)} \text{ and } f(5) = \frac{1}{2}.$$

Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with an error less than $\frac{1}{1000}$.

Multiple Choice

15. Suppose a function f is approximated with a fourth-degree Taylor polynomial about $x = 1$. If the maximum value of the fifth derivative between $x = 1$ and $x = 3$ is 0.01, that is, $|f^{(5)}(x)| < 0.01$, then the maximum error incurred using this approximation to compute $f(3)$ is
- (A) 0.054 (B) 0.0054 (C) 0.26667 (D) 0.02667 (E) 0.00267

16. What are all the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ converges?
- (A) $-1 \leq x \leq 1$ (B) $-1 < x < 1$ (C) $-1 < x \leq 1$ (D) $-1 \leq x < 1$ (E) All real x

17. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin(x^2)$ is

- (A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

18. The maximum error incurred by approximating the sum of the series $1 - \frac{1}{2!} + \frac{2}{3!} - \frac{3}{4!} + \frac{4}{5!} - \dots$ by the sum of the first six terms is

- (A) 0.001190 (B) 0.006944 (C) 0.33333 (D) 0.125000 (E) None of these

19. If f is a function such that $f'(x) = \sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

- (A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

20. Now that you have finished the last question of the last “new concept” worksheet of your high school career, how do you feel? (Show your work)

- (A) Relieved (B) Very Sad (C) Euphoric (D) Tired (E) All of these