Name____ Date_______________________ Period_____

Xtra Credit: Limit Error

(from Dave L. Renfro)

Consider the following limit computations:

$$\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)=\lim_{x\to-\infty}\left[\frac{\left(x+\sqrt{x^2+2x}\right)}{1}\cdot\frac{\left(x-\sqrt{x^2+2x}\right)}{\left(x-\sqrt{x^2+2x}\right)}\right]$$

$$=\lim_{x\to-\infty}\left[\frac{-2x}{x-\sqrt{x^2+2x}}\right]$$

$$=\lim_{x\to-\infty}\left[\frac{-2x}{\left(x-\sqrt{x^2+2x}\right)}\cdot\frac{x^{-1}}{x^{-1}}\right]$$

$$=\lim_{x\to-\infty}\left[\frac{-2}{1-\sqrt{\left(x^{-2}\right)\left(x^2+2x\right)}}\right]$$

$$=\lim_{x\to-\infty}\left[\frac{-2}{1-\sqrt{1+\frac{2}{x}}}\right]$$

As $x\to-\infty$, we can see that $\sqrt{1+\frac{2}{x}}\to1^-$ (approaches 1 from below 1, since $1+\frac{2}{x}$ will be 1 plus a negative number that's almost equal to zero). Therefore, $\left(1-\sqrt{1+\frac{2}{x}}\right)\to0^+$ (approaches zero from above, since 1 minus a number smaller than one but really close to one will be positive but almost equal to zero). So, when x is a large negative number (approaching $-\infty$), $\left(1-\sqrt{1+\frac{2}{x}}\right)$ will be a positive number that's almost equal to zero. That means, then, that the last line shown above involves -2 divided by positive numbers approaching zero, and so this last limit is $\frac{-2}{0^+}=-\infty$. However, the ACTUAL value of $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)$ is -1, and you can prove it to yourself be evaluating $x+\sqrt{x^2+2x}$ on your calculator for large negative numbers. Bear in mind, though, that no amount of numerical verification mathematically PROVES that the limit is -1. Direct substitution gives the indeterminate form $\infty-\infty$.

<u>**Directions**</u>: **On this sheet, answer the following questions. Each part is worth 5 points added to a test score. The two parts may be answered independently, but no partial credit will be given on each part.**

(a) (+5) Find at least one mathematical error in the above calculations that invalidates the result (be very specific) and then…

(b) (+5) Re-do the algebraic calculation correctly (or find an alternative algebraic method that is mathematically valid) to come up with the correct limit of -1 (be very detailed in your steps).