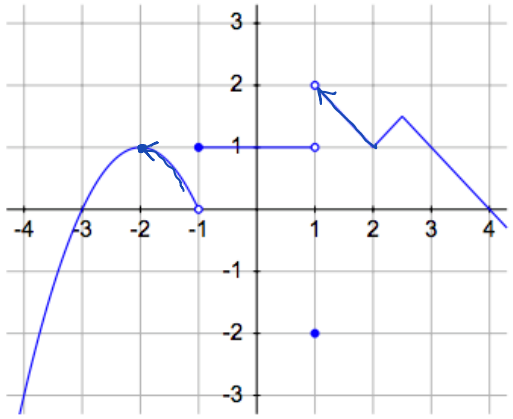
10 checks total

Name KEY Date Friday, 9/14/2018 Breakfast Taco Ingredients Bacon Egg Potato & Cheese = Jorn

AP Calculus TEST: 1.1-1.4—Limits and Continuity. No Calculator. BC

Part I: Multiple Choice—write the CAPITAL LETTER in the blank to the left of the problem number.

Use the graph of the function $f(x)$ shown at right to answer questions 1.



A 1. $\lim_{x\to -2^+} f(f(x)) =$ $f(\lim_{x\to -2^+} f(x)) \neq f(1)$; $\lim_{x\to 1^-} f(x) = 1$ ← From below = From left

(A) 1 (B) 2 (C) -2 (D) 0 (E) No such value exists

C 2. $\lim_{x\to 0} \frac{(2+x)^2+3(2+x)-10}{x} \overset{0/0}{=} \lim_{x\to 0} \frac{4+4x+x^2+6+3x-10}{x} = \lim_{x\to 0} \frac{x^2+7x}{x} = \lim_{x\to 0} \frac{x(x+7)}{x} = 0+7 = 7$

(A) -3 (B) 0 (C) 7 (D) 9 (E) No such value exists

B 3. $\lim_{x\to -3^-} \frac{(x+1)(x+4)}{(x+3)(x+1)} =$ $\frac{1}{0} = \frac{\neq 0}{0} \to VA \to \pm\infty$

plug in -3.1 for sign of ∞: $\frac{+}{-} = -$ → $-\infty$

(A) ∞ (B) $-\infty$ (C) $\frac{7}{18}$ (D) $\frac{4}{3}$ (E) 1

E 4. $f(x) = \begin{cases} 3x+10, & x<-2 \\ \frac{2x}{x+1}, & -2\le x<1 \\ \sqrt{x-1}, & x\ge 1 \end{cases}$ VA @ $x=-1 \in [-2,1)$

At $x=-2$: $\lim_{x\to -2^-} f(x) = 4$; $\lim_{x\to -2^+} f(x) = f(-2) = 4$; $4=4$, so, f is cont. @ $x=-2$

At $x=1$: $\lim_{x\to 1^-} f(x) = 1$; $\lim_{x\to 1^+} f(x) = f(1) = 0$; $1\neq 0$, so, f is not cont. @ $x=1$

Let $f(x)$ be defined by the piecewise equation above, then $f(x)$ is continuous

(A) for all $x\neq -2,-1$ (B) for all $x\neq -1$ (C) for all $x\neq 1$ (D) for all $x\neq -2,\ 1$ (E) for all $x\neq -1,1$

A 5. $\lim_{x\to 1} \frac{\frac{2}{x+3}-\frac{1}{2}}{x-1} \overset{0/0}{=}$ (LCM / LCM) $= \frac{2(x+3)}{2(x+3)}$

(A) $-\frac{1}{8}$ (B) $\frac{1}{8}$ (C) DNE (D) $\frac{1}{6}$ (E) $-\frac{1}{6}$

$\lim_{x\to 1} \frac{2(2)-(x+3)}{2(x-1)(x+3)}$; $\lim_{x\to 1} \frac{4-x-3}{2(x-1)(x+3)}$; $\lim_{x\to 1} \frac{1-x}{2(x-1)(x+3)}$

$\lim_{x\to 1} \frac{-1(x-1)}{2(x-1)(x+3)}$; $\frac{-1}{2(1+3)}$; $-\frac{1}{8}$

E 6. Evaluate $\lim_{x\to 0}\left(\frac{4\tan 3x}{3\tan 4x}-\frac{\cos 2x-1}{2x}+\frac{\sin 6x}{3x}+\sec x\right)=$ (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$\frac{0}{0}$ $\frac{0}{0}$ $\frac{0}{0}$ direct sub

$\frac{4}{3}\left(\frac{3}{4}\right)-0+\frac{6}{3}+\sec 0$

$1-0+2+1$

4

B 7. If $f(x)=\begin{cases}\frac{\sqrt{x-2}-2}{x-6}, & x\neq 6\\ k, & x=6\end{cases}$, find the value of k that makes $f(x)$ continuous at $x=6$.

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$ (E) $\frac{1}{6}$

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$\lim_{x\to 6}\frac{\sqrt{x-2}-2}{x-6}\cdot\frac{\sqrt{x-2}+2}{\sqrt{x-2}+2}$ $\frac{0}{0}$

$\lim_{x\to 6}\frac{x-2-4}{(x-6)(\sqrt{x-2}+2)}$

$\lim_{x\to 6}\frac{(x-6)\cdot 1}{(x-6)(\sqrt{x-2}+2)}$

$\frac{1}{\sqrt{6-2}+2}$

$\frac{1}{4}$

E 8. $\lim_{x\to -1}\frac{x^3-3x^2-3x+1}{x^2-x-2}\overset{\frac{0}{0}}{=}$

(A) $-\frac{2}{3}$ (B) $-\frac{4}{3}$ (C) ∞ (D) 4 (E) -2

Synthetic division on numerator:

	1	−3	−3	1
−1		−1	4	−1
	1	−4	1	0

So, $\lim_{x\to -1}\frac{(x+1)(x^2-4x+1)}{(x+1)(x-2)}$

$\frac{1+4+1}{-1-2}$

$\frac{6}{-3}$

-2

E 9. The function $g(x)=\frac{3x^3-x^2+4x-5}{x^2+2x-2}$ has an end behavior asymptote of which of the following?

(A) $y=3$ (B) $y=-3$ (C) $y=3x$ (D) $y=3x+5$ (E) $y=3x-7$

$$\begin{array}{r} 3x-7 \\ x^2+2x-2\overline{)\,3x^3-x^2+4x-5} \\ \underline{-3x^3+6x^2+6x} \\ -7x^2+10x-5 \end{array}$$

So, g has a Slant Asymptote (SA) at $y=3x-7$

Part II: Free Response: Show all work in the space provided. Be sure to use proper notation, notation, notation. No notation, No no points!!!

Let $f(x)=\begin{cases} \dfrac{-2x^3-5x^2+4x-9}{\sqrt{10x^6+7x^4+8x^2+1}}, & x\le -100 \\ \dfrac{(3+x)^2-3(3+x)-28}{x+7}, & -100< x\le -5 \\ \dfrac{5x^3}{\sin^3 2x}, & -5<x\le 1 \\ bx^2+a, & 1<x<2 \\ 7, & x=2 \\ 5bx-a, & 2<x<7 \\ \sqrt{x+2}, & 7\le x<14 \\ \dfrac{1}{2}x-2, & x\ge 14 \end{cases}$

(a) Find $\lim_{x\to -7} f(x)$

$\lim_{x\to -7} \dfrac{(3+x)^2-3(3+x)-28}{x+7}$ $\frac{0}{0}$

$\lim_{x\to -7} \dfrac{9+6x+x^2-9-3x-28}{x+7}$

$\lim_{x\to -7} \dfrac{x^2+3x-28}{x+7}$ $\frac{0}{0}$

$\lim_{x\to -7} \dfrac{(x+7)(x-4)}{(x+7)}$

$\lim_{x\to -7} (x-4)$

$-7-4$

-11 ✓

(b) Find $\lim_{x\to -\infty} f(x)$

$\lim_{x\to -\infty} \dfrac{-2x^3-5x^2+4x-9}{\sqrt{10x^6+7x^4+8x^2+1}} \approx \dfrac{-2x^3}{\sqrt{10}\,x^3} \to \pm\dfrac{2}{\sqrt{10}}$

✓ $+\dfrac{2}{\sqrt{10}}$ ✓ ← (or $\frac{2\sqrt{10}}{10}=\frac{\sqrt{10}}{5}$)

* plug in a neg into $\dfrac{-2x^3}{\sqrt{10x^6}}$ for sign: $\frac{+}{+}=+$

(c) $\lim_{x\to 0} f(x)=$

$\lim_{x\to 0} \dfrac{5x^3}{\sin^3 2x}$

$\lim_{x\to 0} (5)\left(\dfrac{1x}{\sin 2x}\right)^3$

$5\left(\frac{1}{2}\right)^3$

$\frac{5}{2^3}$

$\frac{5}{8}$ ✓

(d) Find the values of a and b that make f continuous at $x=2$. Show all steps, and use correct notation, notation, notation.

$\lim_{x\to 2^-} f(x) = 4b+a$

$f(2) = 7$

$\lim_{x\to 2^+} f(x) = 10b-a$

So, $4b+a=7$

$+\quad 10b-a=7$

$14b=14$

$b=1$ ✓

So, $4(1)+a=7$

$a=7-4$

$a=3$ ✓

(e) Using the 3-step definition of continuity at a point, determine if $f(x)$ is continuous at $x=14$. Justify.

$\lim_{x\to 14^-} f(x) = \sqrt{14+2} = 4$ ✓

$f(14) = 7-2 = 5$

$\lim_{x\to 14^+} f(x) = 5$

✓ either one of these

f is not continuous at $x=14$ since $4\neq 5$ ✓

9 checks

18 total for test!