Name KEY Date____________ Obscure Constellation______________

AB Calculus TEST: 2.1 - 2.6, NO CALCULATOR

Multiple Choice—Put the correct CAPITAL letter in the space on the OFFICIAL answer sheet of the NFL.

T 1.

$f'(x) = \frac{(x^2+c^2)(2x)-(x^2-c^2)(2x)}{(x^2+c^2)^2}$ $f'(x) = \frac{2x(2c^2)}{(x^2+c^2)^2}$

$f'(x) = \frac{2x(x^2+c^2-x^2+c^2)}{(x^2+c^2)^2}$ $f'(x) = \frac{4c^2x}{(x^2+c^2)^2}$

If $f(x) = \dfrac{x^2 - c^2}{x^2 + c^2}$ where c is a constant, find $f'(x)$.

T) $\dfrac{4c^2x}{(x^2+c^2)^2}$ V) $\dfrac{-4c^2x}{(x^2+c^2)^2}$ W) $\dfrac{4cx(c-x)}{(x^2+c^2)^2}$ X) $\dfrac{x-c}{x+c}$ Y) 0

L 2.

If $f(x) = x^2 - x$ and $g(x) = \dfrac{1}{x}$, and $h(x)$ is defined with the expressions below, arrange the values of $h'(1)$ from smallest to largest.

$h'(x) = 2x - 1 + \frac{1}{x^2}$ $h(x) = \frac{x^2-x}{x} = x-1$, $h' = \boxed{1}$ $h(x) = \frac{1}{x^2} - \frac{1}{x}$, $h'(x) = \frac{-2}{x^3} + \frac{1}{x^2}$, $h'(1) = -2+1 = \boxed{-1}$

I. $h(x) = f(x) - g(x)$ II. $h(x) = f(x)g(x)$ III. $h(x) = f(g(x))$

$h'(1) = 2 - 1 + 1 = 2$

G) I, II, III H) I, III, II J) II, III, I K) III, I, II L) III, II, I

I 3. (Note $\Delta x = h$)

If $f(x) = x^3 + 2x^2 - 5x - 1$, find $\lim\limits_{\Delta x \to 0} \dfrac{f'(-3+\Delta x) - f'(-3)}{\Delta x}$.

$f' = 3x^2 + 4x - 5$

$f'' = 6x + 4$

$f''(-3) = -18 + 4 = -14$

A) 10 E) 20 I) −14 O) −18 U) does not exist

T 4.

$f' = \frac{-2}{(x-1)^2}$

If $f(x) = \dfrac{x+1}{x-1}$, which of the following statements is true? $f'' = \frac{4}{(x-1)^3}$

$x-1 \overline{)x+1} = 1 + \frac{2}{x-1}$

I. $\lim\limits_{x\to 1} f(x) = -\infty$ II. $\lim\limits_{x\to 1} f'(x) = -\infty$ III. $\lim\limits_{x\to 1} f''(x) = -\infty$

S) I only T) II only V) III only W) I and II only X) II and III only

O 5.

Let $f(x) = \sin x \cos x + x$ for $0 \le x \le \dfrac{3\pi}{2}$. Find all values for which $f'(x) = 1$.

$f' = \cos^2 x - \sin^2 x + 1$

$f' = \cos 2x + 1 = 1$

$\cos 2x = 0$

$2x = \frac{\pi}{2}, x = \frac{\pi}{4}$

$2x = \frac{3\pi}{2}, x = \frac{3\pi}{4}$

$2x = \frac{5\pi}{2}, x = \frac{5\pi}{4}$

I. $x = \dfrac{\pi}{4}$ II. $x = \dfrac{3\pi}{4}$ III. $x = \dfrac{5\pi}{4}$

A) I only B) II only I) III only O) I, II, and III E) I and III only

N 6.

If $f(x) = \sec x$ and $k = \dfrac{7\pi}{4}$, arrange i) $f(k)$ ii) $f'(k)$ iii) $f''(k)$ from smallest to largest

$f(\frac{7\pi}{4}) = \sqrt{2}$

F) i, ii, iii L) i, iii, ii N) ii, i, iii R) ii, iii, i S) iii, i, ii

$f' = \sec x \tan x$

$f'(\frac{7\pi}{4}) = \sqrt{2}(-1) = -\sqrt{2}$

$f'' = \sec x \tan^2 x + \sec^3 x$

$f''(\frac{7\pi}{4}) = \sqrt{2}(1) + 2\sqrt{2} = 3\sqrt{2}$

U 7. (Note: $\Delta x = h$)

Find $\lim_{\Delta x \to 0} \dfrac{\csc\left(\frac{\pi}{6}+\Delta x\right)-\csc\frac{\pi}{6}}{\Delta x}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$f' = -\csc x \cot x$

$f'\left(\frac{\pi}{6}\right) = (-2)(\sqrt{3}) = -2\sqrt{3}$

A) $\dfrac{-2\sqrt{3}}{3}$ E) $\dfrac{2\sqrt{3}}{3}$ I) $\dfrac{\sqrt{3}}{2}$ O) $2\sqrt{3}$ U) $-2\sqrt{3}$

C 8.

If $h(x) = \dfrac{f(x)-g(x)}{x+f(x)}$, find $h'(1)$

$h' = \dfrac{(x+f)(f'-g')-(f-g)(1+f')}{(x+f)^2}$

$h'(1) = \dfrac{(1+4)(5)-(6)(3)}{25}$

$= \dfrac{25-18}{25} = \dfrac{7}{25}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	2	-2	-3

A. $\dfrac{2}{3}$ B. $\dfrac{5}{3}$ C. $\dfrac{7}{25}$ D. $\dfrac{13}{25}$ E. $\dfrac{-31}{25}$

B 9.

The line $y = 9x + 16$ is tangent to $f(x) = x^3 - 3x$ at

$m = 9$

$f' = 3x^2 - 3 = 9$

$3x^2 = 12$

$x^2 = 4$

$\boxed{x = \pm 2}$

I. $x = 2$ II. $x = -2$ III. $x = 3$

test y-values

$y(2) = 18 + 16 = 34$

$f(2) = 2$

$34 \neq 2$

$y(-2) = -18 + 16 = -2$

$f(-2) = -8 + 6 = -2$

$-2 = -2$ ✓

A. I only B. II only C. III only D. I and II only E. II and III only

C 10.

If $f(x) = \sqrt{x}\cot x$, find $f'(x)$.

$f' = \left(\frac{1}{2}x^{-1/2}\right)(\cot x) + \left(x^{1/2}\right)(-\csc^2 x)$

$= \frac{1}{2}x^{-1/2}\left[\cot x - 2x\csc^2 x\right]$

$= \dfrac{\cot x - 2x\csc^2 x}{2\sqrt{x}}$

A. $\dfrac{\cot x}{2\sqrt{x}}$ B. $-\sqrt{x}\csc^2 x$ C. $\dfrac{\cot x - 2x\csc^2 x}{2\sqrt{x}}$

D. $\dfrac{\cot x - 2x\csc x\cot x}{2\sqrt{x}}$ E. $\dfrac{\cot x + 2x\csc x\cot x}{2\sqrt{x}}$

D 11.

The slope of the line normal to $f(x) = \dfrac{\tan x}{x}$ at $x = \dfrac{\pi}{4}$ is

$f' = \dfrac{(x)\sec^2 x - \tan x}{x^2}$

$f'\left(\frac{\pi}{4}\right) = \dfrac{\left(\frac{\pi}{4}\right)(2) - 1}{\frac{\pi^2}{16}}$

$= \left(\frac{\pi}{2} - 1\right)\left(\frac{16}{\pi^2}\right) = \frac{8}{\pi} - \frac{16}{\pi^2} = \frac{8\pi - 16}{\pi^2}$, $m_N = \dfrac{-\pi^2}{8\pi - 16}$

A. $\dfrac{-1}{2}$ B. $\dfrac{2\pi - 4}{\pi}$ C. $\dfrac{\pi}{4 - 2\pi}$ D. $\dfrac{-\pi^2}{8\pi - 16}$ E. $\dfrac{8\pi - 16}{\pi^2}$

D 12.

If $f(x) = \sin(\sin(\sin \pi x))$, find $f'(1)$.

$f' = \cos(\sin(\sin \pi x))\cos(\sin \pi x) \cdot \cos \pi x \cdot \pi$

$f'(1) = (1)(1)(-1)(\pi) = -\pi$

A. 0 B. -1 C. 1 D. $-\pi$ E. π

$\boxed{A}$ 13.

If $f(x)=\begin{cases} ax^3+x^2-4, x<2 \\ b(x-3)^2+4x, x\geq 2 \end{cases}$, where a and b are constants, find the value of a that makes f differentiable.

Cont
$8a=b+8$
$b=8a-8$

$f'=\begin{cases} 3ax^2+2x, x<2 \\ 2b(x-3)+4, x>2 \end{cases}$

Slope
$12a+4=-2b+4$
$12a+\cancel{4}=-2(8a-8)\cancel{+4}$
$12a=-16a+16$
$28a=16$
$a=\frac{16}{28}=\frac{4}{7}$

A. $\frac{4}{7}$ B. 4 C. –4 D. $a=\frac{1}{2}$ E. $\frac{-24}{7}$

$\boxed{D}$ 14.

Let the functions f and g and their derivatives be defined at $x=1$ and $x=-1$ by the table below. If $f(g(x))$ has a horizontal tangent at $x=1$, find the value of k.

deriv $f'(g(x))\cdot g'(x)$

deriv = 0 when x = 1: $f'(g(1))\cdot g'(1)=0$
$f'(-1)\cdot(k^3-4k)=0$
$\frac{k^3-4k}{k}=0$
$k^3-4k=0$
$k(k^2-4)=0$
$\cancel{k=0}, -2, 2$
$\frac{0}{0}$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	–1	6	k^3-4k
–1	–2	$-k$	$\frac{1}{k}$	0

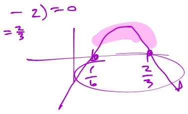
I. $k=0$ II. $k=2$ III. $k=-2$

A. I only B. II only C. III only D. II and III only E. I, II, and III

$\boxed{A}$ 15.

A particle moves along the x-axis such that at any time $t\geq 0$, its position function is given by $x(t)=-12t^3+15t^2-4t+5$. For what values of t is the particle moving to the right? , $v>0$

$x'(t)=v(t)=-36t^2+30t-4$
$-2(18t^2-15t+2)=0$
$-2(6t-1)(3t-2)=0$
$t=\frac{1}{6}, t=\frac{2}{3}$

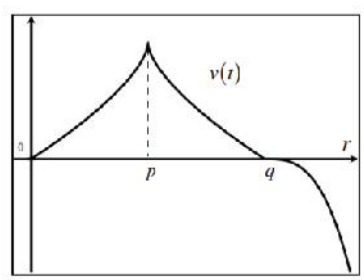


A. $\frac{1}{6}<t<\frac{2}{3}$ B. $0\leq t<\frac{1}{6}, t>\frac{2}{3}$ C. $0\leq t<\frac{12}{5}$

D. $t>\frac{12}{5}$ E. $t>\frac{2}{3}$

$\boxed{E}$ 16.

A particle travels on a straight line with velocity $v(t)$ as given in the figure to the right. In what interval(s) is the particle speeding up?



I. $(0,p)$ ✓ II. (p,q) III. (q,r) ✓

A. I only B. II only C. III only
D. I and II only E. I and III only

$\boxed{D}$ 17.

A particle moves along the x-axis so that at any time t its position is given by $x(t)=-4\cos\frac{\pi t}{2}-\pi t, t\geq 0$.

What are all values of t, $0\leq t\leq 2$, for which the particle is at rest?

$x'(t)=v(t)=4\sin\left(\frac{\pi}{2}t\right)\cdot\frac{\pi}{2}-\pi=0$
$2\pi\sin\left(\frac{\pi}{2}t\right)=\pi$
$\sin\left(\frac{\pi}{2}t\right)=\frac{1}{2}$
$\frac{\pi}{2}t=\frac{\pi}{6}, t=\frac{1}{3}$
$\frac{\pi}{2}t=\frac{5\pi}{6}, t=\frac{5}{3}$

A. $t=\frac{1}{6}, t=\frac{11}{6}$ B. $t=\frac{1}{6}, t=\frac{5}{6}$ C. $t=\frac{1}{2}, t=\frac{3}{2}$ D. $t=\frac{1}{3}, t=\frac{5}{3}$ E. $t=1$

I 18.

$y' = 2x(1-2x)^3 + x^2(3)(1-2x)^2(-2)$

$y' = x(1-2x)^2[2(1-2x) - 6x]$

$y' = x(1-2x)^2(2-10x)$

$y' = 2x(1-2x)^2(1-5x)$

$y' = -2x(1-2x)^2(5x-1)$

If $y = x^2(1-2x)^3$, find y'.

A) $-x(2x-1)^2(x-2)$ E) $2x(1-2x)^2(5x-1)$ I) $-2x(2x-1)^2(5x-1)$

O) $3x^2(2x-1)^2$ U) $-6x^2(1-2x)^2$

T 19.

If $f(x) = \sin^2\left(2x + \frac{\pi}{4}\right)$, find $\lim_{x\to\pi/4} \frac{f(x) - f(\pi/4)}{x - \pi/4}$.

$f' = 2\left(\sin\left(2x+\frac{\pi}{4}\right)\right)\cdot\cos\left(2x+\frac{\pi}{4}\right)\cdot(2)$

$f'\left(\frac{\pi}{4}\right) = 4\cdot\sin\left(\frac{3\pi}{4}\right)\cdot\cos\left(\frac{3\pi}{4}\right)$

$= 4\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= -2$

R) $\frac{-\sqrt{2}}{2}$ S) $\frac{\sqrt{2}}{2}$ T) -2 V) 2 W) -1

H 20.

In the table below, the values of $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$ are given for two values of x. If $y = [f(2x) + g(x)]^2$, find $y'(3)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	4	−1	2	−5
6	−3	5	−2	−4

$y' = 2(f(2x) + g(x))(2f'(2x) + g'(x))$

$y'(3) = 2(-3 + -1)(2(-2) + -5)$

$= 2(-4)(-9) = 72$

C) 0 D) −8 F) −12 G) 56 H) 72

C 21.

If $f(3) = \frac{5\pi}{3}$ and $f'(3) = \sqrt{3}$, find the equation of the line tangent to $g(x) = \cos(f(x))$ at $x = 3$.

$g'(x) = -\sin(f(x))\cdot f'(x)$

$g'(3) = -\sin(f(3))\cdot f'(3)$

$g'(3) = (-\sqrt{3})\sin\left(\frac{5\pi}{3}\right)$ $\frac{5\pi}{3}: \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$g'(3) = (-\sqrt{3})\left(-\frac{\sqrt{3}}{2}\right)$

$g'(3) = \frac{3}{2} = m$

$g(3) = \cos(f(3))$

$= \cos\frac{5\pi}{3}$

$g(3) = \frac{1}{2}$ eq: $y = \frac{1}{2} + \frac{3}{2}(x-3)$

$2y = 1 + 3(x-3)$

$2y = 1 + 3x - 9$

$2y = 3x - 8$

$3x - 2y = 8$

A. $\sqrt{3}x + 2y = 3\sqrt{3} + 1$ B. $\sqrt{3}x + 2y = 3\sqrt{3} - 1$ C. $3x - 2y = 8$

D. $3x + 2y = 8$ E. $x + 2y = 3 + 3\sqrt{3}$

E 22.

If $h(x) = [f(g(x))]^2$, use the chart below to find $h'(2)$.

$h'(x) = 2[f(g(x))]'\cdot f'(g(x))\cdot g'(x)$

$h'(2) = 2[f(-2)]\cdot f'(-2)\cdot(4)$

$= (2)(5)(\pi)(4)$

$= 40\pi$

	f	g	f'	g'
2	−1	−2	3	4
−2	5	−4	π	2

A. 20 B. 2π C. 8π D. 20π E. 40π

D 23.

The 30[th] derivative of $\cos 2x$ is

$y^{0} = \cos 2x$

$y' = -2\sin 2x$

$y'' = -2^2\cos 2x$

$y''' = 2^3\sin 2x$

$y^{(4)} = 2^4\cos 2x$

$4\overline{)30}$ 7, 28, R2

$-2^{30}\cos 2x$

A. $60\sin 2x$ B. $-60\sin 2x$ C. $-60\cos 2x$ D. $-2^{30}\cos 2x$ E. $2^{30}\cos 2x$

B 24. $5 + 5 =$ A. 11 B. 10 C. 9 D. 9.9999999 E. 10.000000001

D 25. November falls immediately between October and which month?

A. July B. Monday C. Decimeter D. December E. Wurstfest

Name____ Date________________ Period____________

AB Calculus TEST: 2.1 - 2.6, NO CALCULATOR

OFFICIAL COVER SHEET OF THE NFL

Put the CAPITAL letter of the correct answer in the space provided. Attach all work to separate paper. No credit will be given if no work is provided.

T 1.	B 9.	D 17.
L 2.	C 10.	I 18.
I 3.	D 11.	T 19.
T 4.	D 12.	H 20.
O 5.	A 13.	C 21.
N 6.	D 14.	E 22.
U 7.	A 15.	D 23.
C 8.	E 16.	B 24.
		D 25.

I certify that I received no help on this take-home test. All answers were obtained through my own efforts and industry without a calculator. I did not get answers from another, nor did I cheat in any way. I am prepared to answer and justify, in person, any answer to any question I get correct, and I will willingly do so, because I am an impeccably honest individual with the utmost integrity and nothing to hide. I love math, too!

____ ____

(sign your name above) (date of signature)