Name KEY Date_________ Favorite Polka Song________

AP Calculus TEST: 2.1-2.8, NO CALCULATOR

Part I: Multiple Choice—Put the correct CAPITAL letter in the space to the left of each question.

C 1. For $x^2y + \sec y = 8$, what is $\frac{dy}{dx}$?

(A) $-2xy\left(x^2 \sec y \tan y\right)$ (B) $\frac{x^2 y}{\sec y \tan y}$ (C) $\frac{-2xy}{x^2 + \sec y \tan y}$ (D) $\frac{-2xy}{x^2 - \sec y \tan y}$

$\frac{d}{dx}: 2xy + x^2\frac{dy}{dx} + \sec y \tan y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^2 + \sec y \tan y) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2 + \sec y \tan y}$

A 2. What are the y coordinates of the points where $y^3 + x^2 + 30y^2 = 4x + 3$ has vertical tangents?

(A) $y = 0$ and $y = -20$ (B) $y = 2$ and $y = -20$ (C) $y = 0$ and $y = 2$ (D) $y = -20$

$\frac{d}{dx}: 3y^2\frac{dy}{dx} + 2x + 60y\frac{dy}{dx} = 4$

$\frac{dy}{dx}(3y^2 + 60y) = 4 - 2x$

$\frac{dy}{dx} = \frac{4-2x}{3y^2+60y}$

$\frac{dy}{dx} = \pm\infty$ when $3y^2 + 60y = 0$

$3y(y+20) = 0$

$y = 0, y = -20$

B 3. Given $xy^2 = 4$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(1,2)$?

(A) $\frac{3}{4}$ (B) $\frac{3}{2}$ (C) 1 (D) $\frac{1}{2}$

$\frac{d}{dx}: (1)y^2 + x \cdot 2y\frac{dy}{dx} = 0$

$2xy\frac{dy}{dx} = -y^2$

$\frac{dy}{dx} = \frac{-y^2}{2xy}$

$\frac{dy}{dx} = \frac{-y}{2x}$

$\frac{d}{dx}: \frac{d^2y}{dx^2} = \frac{(2x)(-\frac{dy}{dx}) - (-y)(2)}{(2x)^2}$

$= \frac{2x\left(-\left(-\frac{y}{2x}\right)\right) + 2y}{4x^2}$

$= \frac{y + 2y}{4x^2}$

$= \frac{3y}{4x^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{3(2)}{4(1^2)}$

$= \frac{3}{2}$

D 4. $\lim_{h \to 0} \frac{\left[4\cos^4(x+h) + 3\sin(x+h)\right] - \left[4\cos^4 x + 3\sin x\right]}{h} =$ $f'(x)$, for some $f(x)$

(A) $16\cos^3 x \sin x + 3\cos x$ (B) $-4\cos^3 x \sin x + 3\cos x$

(C) $-16\cos^3 x + 3\cos x$ (D) $-16\cos^3 x \sin x + 3\cos x$

$f(x) = 4(\cos x)^4 + 3\sin x$

$f'(x) = 16(\cos x)^3(-\sin x) + 3\cos x$

$= -16\cos^3 x \sin x + 3\cos x$

D ______ 5. What is the equation of the <u>normal line</u> to the graph of $y = \cot x$ at the point where $x = \frac{\pi}{2}$?

(A) $y - 1 = -x + \frac{\pi}{2}$ (B) $y = \frac{\pi}{2}x - 1$ (C) $y = -x + \frac{\pi}{2}$ (D) $y = x - \frac{\pi}{2}$

y-value: $y(\frac{\pi}{2}) = \cot\frac{\pi}{2} = 0$
So, point is $(\frac{\pi}{2}, 0)$

$y' = -\csc^2 x$
tangent slope: $y'(\frac{\pi}{2}) = -(\csc\frac{\pi}{2})^2 = -1$
normal slope $= -\frac{1}{y'(\frac{\pi}{2})} = 1$ (opp recip)

Normal line: $y = 0 + 1(x - \frac{\pi}{2})$
$y = x - \frac{\pi}{2}$

B ______ 6. What is the derivative of $\frac{4x^2 - 3x + 7}{5x}$?

(A) $\frac{4x^2+7}{25x^2}$ (B) $\frac{4x^2-7}{5x^2}$ (C) $\frac{7-4x^2}{5x^2}$ (D) $\frac{4x-8}{25x^2}$

$\frac{d}{dx}$: $\frac{(5x)(8x-3) - (4x^2-3x+7)(5)}{(5x)^2}$

$\frac{5[8x^2 - 3x - 4x^2 + 3x - 7]}{25x^2}$

$\frac{4x^2-7}{5x^2}$

C ______ 7. If $f(x) = \sqrt{6\sin x + 9}$, then $f'(0) =$

(A) $\frac{1}{2\sqrt{3}}$ (B) 0 (C) 1 (D) $\frac{\sqrt{3}}{6}$

$f(x) = (6\sin x + 9)^{1/2}$
$f'(x) = \frac{1}{2}(6\sin x + 9)^{-1/2} \cdot (6\cos x)$
$f'(x) = \frac{3\cos x}{\sqrt{6\sin x + 9}}$
$f'(0) = \frac{3\cos 0}{\sqrt{6\sin 0 + 9}}$
$= \frac{3}{3}$
$= 1$

A ______ 8. If $y = \arctan\left(\frac{x}{2}\right)$, then $\frac{dy}{dx} =$

(A) $\frac{2}{x^2+4}$ (B) $\frac{1}{2+x^2}$ (C) $\frac{4}{4+x^2}$ (D) $\frac{4}{2+x^2}$

$y' = \frac{1}{1 + (\frac{x}{2})^2} \cdot \left(\frac{1}{2}\right)$
$= \frac{1}{(1 + \frac{x^2}{4})2}$
$= \frac{1}{2 + \frac{x^2}{2}}\left(\frac{2}{2}\right)$
$= \frac{2}{4 + x^2}$
$= \frac{2}{x^2+4}$

C
______ 9. If $g(x)=\sin^{-1}(x)-\sqrt{1-x^2}$, then $g'(x)=$

(A) $\dfrac{1}{2\sqrt{1-x^2}}$ (B) $\dfrac{2}{\sqrt{1-x^2}}$ (C) $\dfrac{1+x}{\sqrt{1-x^2}}$ (D) $\dfrac{x^2}{\sqrt{1-x^2}}$

$g(x) = \sin^{-1}x-(1-x^2)^{1/2}$

$g'(x) = \dfrac{1}{\sqrt{1-x^2}} - \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)$

$= \dfrac{1}{\sqrt{1-x^2}} + \dfrac{x}{\sqrt{1-x^2}}$

$= \dfrac{1+x}{\sqrt{1-x^2}}$

A
______ 10. The position of an object moving along a horizontal line at time t is described by the function $x(t)=-(t^2-2t+4)(t^3-2t)$. What is the object's velocity at time $t=1$?

(A) -3 (B) -2 (C) -1 (D) 1

$x'(t)=v(t)=-\left[(2t-2)(t^3-2t)+(t^2-2t+4)(3t^2-2)\right]$

$v(1)=-\left[0(-1)+(3)(1)\right]$

$=-3$

C
______ 11. Given $f(x)=x^3-3x^2+2x-7$ and $f(g(x))=x=g(f(x))$, what is $g'(f(3))$?

f & g are inverses

(A) $-\dfrac{1}{11}$ (B) 11 (C) $\dfrac{1}{11}$ (D) -11

$f(3)=27-27+6-7$

$=-1$

so, $f: (3,-1)$

$g: (-1,3)$

$f'(x)=3x^2-6x+2$

$f'(3)=27-18+2$

$=11$

$g'(f(3))$

$g'(-1)=\dfrac{1}{f'(3)}$

$=\dfrac{1}{11}$

Part II: Free Response—Show all work in a clear, concise, cogent, an complete manner

12. The table below gives values of the differentiable functions $f(x)$ & $g(x)$ and their derivatives at selected values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	8	−2	6	−3
2	−1	−5	4	6
4	0	4	2	k

(a) If the function $h(x)$ is given by $h(x)=\dfrac{f(x)+g(x)}{g(x)}$, find $h'(1)$.

$$h'(x) = \frac{g[f'+g']-[f+g]g'}{g^2} \quad \sqrt{1}$$

$$h'(1) = \frac{g(1)[f'(1)+g'(1)]-[f(1)+g(1)]g'(1)}{[g(1)]^2}$$

$$= \frac{(6)[-2+-3]-[8+6](-3)}{6^2}$$

$$= \frac{6(-5)-14}{36} = \frac{-30-14(-3)}{36} = \frac{-30+42}{36} = \frac{12}{36} = \frac{1}{3} \quad \sqrt{4}$$

(b) If the function $P(x)$ is given by $P(x)=f(x)g(x)$, find the difference between the instantaneous rate of change of $P(x)$ at $x=2$ and the average rate of change of $P(x)$ between $x=1$ and $x=2$.

$P'(x) = f'g + fg'$

$P'(2) = f'(2)g(2)+f(2)g'(2)$

$= (-5)(4)+(-1)(6)$

$= -20-6$

$= -26 \quad \sqrt{3}$

Avg ROC $= \dfrac{P(2)-P(1)}{2-1}$

$= \dfrac{f(2)g(2)-f(1)g(1)}{1}$

$= (-1)(4)-(8)(6)$

$= -4-48$

$= -52 \quad \sqrt{4}$

So, $-26-(-52)$

$-26+52$

$26 \quad \sqrt{5}$

(c) If $R(x)=\sqrt{x}\cdot g(x)$ and $R'(4)=\pi$, find the value of k.

$R(x) = (x^{1/2})(g(x))$

$R'(x) = (\frac{1}{2}x^{-1/2})(g(x)) + (x^{1/2})(g'(x)) \quad \sqrt{6}$

$R'(x) = \dfrac{g(x)}{2\sqrt{x}} + \sqrt{x}\cdot g'(x)$

$R'(4) = \dfrac{g(4)}{2\sqrt{4}} + \sqrt{4}\, g'(4)$

$R'(4) = \dfrac{2}{2\cdot 2} + 2\cdot k = \pi \quad \sqrt{7}$

$\frac{1}{2} + 2k = \pi$

$2k = \pi - \frac{1}{2}$

$$\boxed{k = \frac{\pi-\frac{1}{2}}{2} = \frac{2\pi-1}{4} = \frac{\pi}{2}-\frac{1}{4} = \frac{1}{2}\left(\pi-\frac{1}{2}\right)} \quad \sqrt{8}$$