Name________________________________ Date______________________ Period_____________

AB Calculus Practice Test: All Rules of Differentiation & Relative extrema

Part I: Multiple Choice

1.

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2)=5$ and $f'(5)=-\frac{1}{2}$, then $g'(-2)=$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

2.

If f and g are twice differentiable and if $h(x)=f(g(x))$, then $h''(x)=$

(A) $f''(g(x))[g'(x)]^2+f'(g(x))g''(x)$

(B) $f''(g(x))g'(x)+f'(g(x))g''(x)$

(C) $f''(g(x))[g'(x)]^2$

(D) $f''(g(x))g''(x)$

(E) $f''(g(x))$

3.

If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

4.

If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x})+e^{-x}$

(C) $\cos(e^{-x})-e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

5.

If $f(x)=(x-1)^{\frac{3}{2}}+\dfrac{e^{x-2}}{2}$, then $f'(2)=$

(A) 1 (B) $\dfrac{3}{2}$ (C) 2 (D) $\dfrac{7}{2}$ (E) $\dfrac{3+e}{2}$

6.

$\dfrac{d}{dx}\left(xe^{\ln x^2}\right)=$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

7.

$\lim\limits_{h\to 0}\dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$

(B) $f'(e)$, where $f(x)=\dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln\left(x+e\right)$

(E) $f'(0)$, where $f(x)=\ln x$

8.

If $f(x) = \frac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\frac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\frac{e^{2x}(2x+1)}{x^2}$

(E) $\frac{e^{2x}(2x-1)}{2x^2}$

9.

If $f(x) = \ln\left|x^2-1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2-1}\right|$

(B) $\frac{2x}{\left|x^2-1\right|}$

(C) $\frac{2|x|}{x^2-1}$

(D) $\frac{2x}{x^2-1}$

(E) $\frac{1}{x^2-1}$

10.

If $y = \arctan(e^{2x})$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\frac{2e^{2x}}{1+e^{4x}}$ (C) $\frac{e^{2x}}{1+e^{4x}}$ (D) $\frac{1}{\sqrt{1-e^{4x}}}$ (E) $\frac{1}{1+e^{4x}}$

11.

If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$

12.

The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

13.

If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

14.

If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

15.

If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

16.

$$\frac{d}{dx}\left(2^x\right) =$$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

17.

$$\frac{d}{dx}\ln\left|\cos\left(\frac{\pi}{x}\right)\right| \text{ is}$$

(A) $\frac{-\pi}{x^2\cos\left(\frac{\pi}{x}\right)}$ (B) $-\tan\left(\frac{\pi}{x}\right)$ (C) $\frac{1}{\cos\left(\frac{\pi}{x}\right)}$

(D) $\frac{\pi}{x}\tan\left(\frac{\pi}{x}\right)$ (E) $\frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$

18.

The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

19.

If $f(x) = e^x$, then $\ln\left(f'(2)\right) =$

(A) 2 (B) 0 (C) $\frac{1}{e^2}$ (D) $2e$ (E) e^2

20.

If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

21.

If $f(x)=\left(x^2+1\right)^x$, then $f'(x)=$

(A) $x\left(x^2+1\right)^{x-1}$

(B) $2x^2\left(x^2+1\right)^{x-1}$

(C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2-1\right)+\dfrac{2x^2}{x^2+1}$

(E) $\left(x^2-1\right)^x\left[\ln\left(x^2+1\right)+\dfrac{2x^2}{x^2+1}\right]$

22.

If $f(x)=\left(x^2+1\right)^{(2-3x)}$, then $f'(1)=$

(A) $-\dfrac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\dfrac{3}{2}\ln(2)$ (D) $-\dfrac{1}{2}$ (E) $\dfrac{1}{8}$

23.

$\dfrac{d}{dx}(\arcsin 2x)=$

(A) $\dfrac{-1}{2\sqrt{1-4x^2}}$ (B) $\dfrac{-2}{\sqrt{4x^2-1}}$ (C) $\dfrac{1}{2\sqrt{1-4x^2}}$

(D) $\dfrac{2}{\sqrt{1-4x^2}}$ (E) $\dfrac{2}{\sqrt{4x^2-1}}$

24.

Let $f(x)=\cos\left(\arctan x\right)$. What is the range of f?

(A) $\left\{x\middle|-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}\right\}$ (B) $\{x|0<x\le 1\}$ (C) $\{x|0\le x\le 1\}$

(D) $\{x|-1<x<1\}$ (E) $\{x|-1\le x\le 1\}$

25.

If $y = e^{nx}$, then $\dfrac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n!e^x$

26.

If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

27.

If $y = x^{\ln x}$, then y' is

(A) $\dfrac{x^{\ln x} \ln x}{x^2}$

(B) $x^{1/x} \ln x$

(C) $\dfrac{2x^{\ln x} \ln x}{x}$

(D) $\dfrac{x^{\ln x} \ln x}{x}$

(E) None of the above

28.

If $f(g(x)) = \ln(x^2 + 4)$, $f(x) = \ln(x^2)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\dfrac{1}{\sqrt{x^2 + 4}}$ (B) $\dfrac{1}{x^2 + 4}$ (C) $\sqrt{x^2 + 4}$ (D) $x^2 + 4$ (E) $x + 2$

29.

If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{(x^2-1)}$

(B) $(2x)10^{(x^2-1)}$

(C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$

(E) $x^2(\ln 10)10^{(x^2-1)}$

30.

An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$

(B) $y = -3x + 1$

(C) $y = 2x + 10$

(D) $y = 3x - 1$

(E) $y = 4x + 1$

31.

If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x) + f''(x)$

(B) $f'(x) + (f''(x))^2$

(C) $(f'(x) + f''(x))^2$

(D) $(f'(x))^2 + f''(x)$

(E) $2f'(x) + f''(x)$

32.

For $0 < x < \frac{\pi}{2}$, if $y = (\sin x)^x$, then $\frac{dy}{dx}$ is

(A) $x\ln(\sin x)$

(B) $(\sin x)^x \cot x$

(C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x(x\cos x + \sin x)$

(E) $(\sin x)^x(x\cot x + \ln(\sin x))$

33.

$\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$

(B) $(\ln x)^x$

(C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$

(D) $(\ln x)\left(x^{\ln x - 1}\right)$

(E) $2(\ln x)\left(x^{\ln x}\right)$

BC Free Response:

1. 2001 AB4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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2. 2008 AB6

Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$.

(a) Show that the derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(b) Write an equation for the line tangent to the graph of f at $x = e^2$.

(c) Find the x-coordinate of the point at which $f'(x) = 0$.

(d) Find the x-coordinate of the point at which $f''(x) = 0$.

(e) Find $\lim_{x \to 0^+} f(x)$.