Name____________________________________ Date____________________ Alchemy Secret________

BC Calculus: TEST 5.1 – 6.1, NO CALCULATOR

Part I: Multiple Choice—Show all work on scratch paper and attach to the back.

_____ 1. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$ (B) $-7+f'(4)$ (C) $\int_2^4 f(t)dt$ (D) $\int_2^4 (-7+f(t))dt$ (E) $-7+\int_2^4 f(t)dt$

_____ 2. $\int x\sin(6x)dx=$

(A) $-x\cos(6x)+\sin(6x)+C$ (B) $-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$ (C) $-\frac{x}{6}\cos(6x)+\frac{1}{6}\sin(6x)+C$

(D) $\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$ (E) $6x\cos(6x)-\sin(6x)+C$

_____ 3. Given that $y(1)=-3$ and $\frac{dy}{dx}=2x+y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x=1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

_____ 4. If $\int x^2\cos x dx = f(x)-\int 2x\sin x dx$, then $f(x)=$

(A) $2\sin x+2x\cos x+C$ (B) $x^2\sin x+C$ (C) $2x\cos x-x^2\sin x+C$

(D) $4\cos x-2x\sin x+C$ (E) $(2-x^2)\cos x-4\sin x+C$

_____ 5. If the graph of $y=f(x)$ contains the point $(0,2)$, $\frac{dy}{dx}=\frac{-x}{ye^{x^2}}$ and $f(x)>0$ for all x, then

$f(x)=$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$ (D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

_____ 6. Population y grows according to the equation $\frac{dy}{dt}=ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

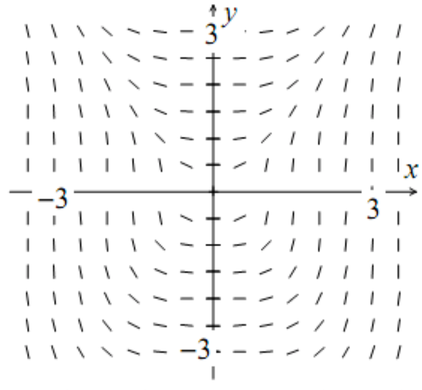
(A) $\ln\sqrt[10]{2}$ (B) $\frac{1}{5}$ (C) $\ln\sqrt{10}$ (D) $2\ln 10$ (E) 5

_____ 7. $\int \frac{x^2}{3+4x+x^2}dx=$

(A) $1+\frac{9}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$ (B) $x-\frac{9}{2}\ln|x+3|+\frac{1}{2}\ln|x+1|+C$ (C) $x+\frac{9}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$

(D) $x-\frac{9}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$ (E) $1+\frac{9}{2}\ln|x+3|+\frac{1}{2}\ln|x+1|+C$

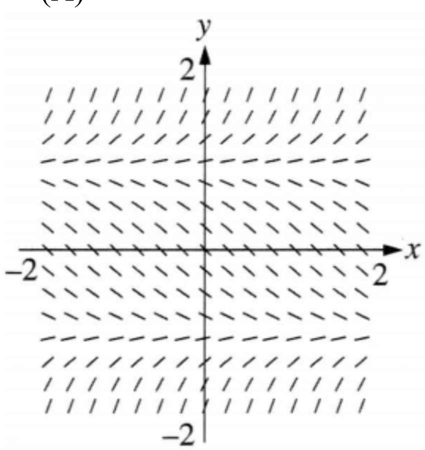
_____ 8. Show below is a slope field for which of the following differential equations?



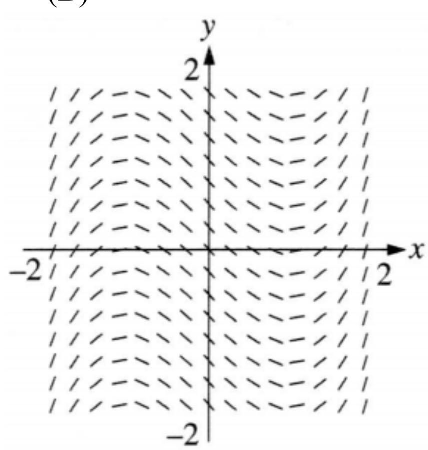
(A) $\dfrac{dy}{dx}=\dfrac{x}{y}$ (B) $\dfrac{dy}{dx}=\dfrac{x^2}{y^2}$ (C) $\dfrac{dy}{dx}=\dfrac{x^3}{y}$ (D) $\dfrac{dy}{dx}=\dfrac{x^2}{y}$ (E) $\dfrac{dy}{dx}=\dfrac{x^3}{y^2}$

_____ 9. Which of the following could be the slope field for the differential equation $\dfrac{dy}{dx}=y^2-1$?

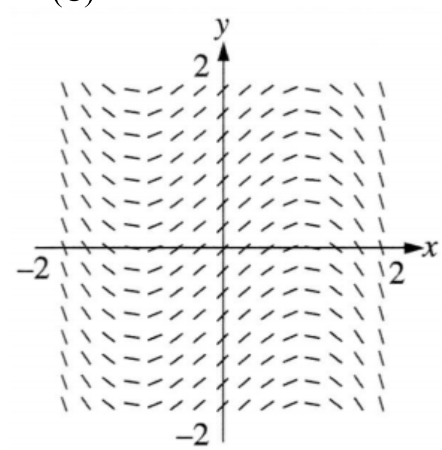
(A)



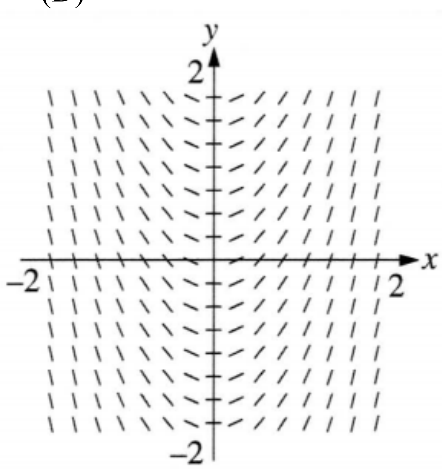
(B)



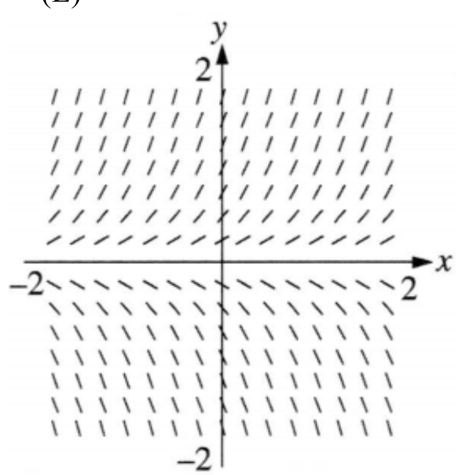
(C)



(D)



(E)



_____ 10. $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^2}\sin\left(t^3\right)dt\right)=$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$ (D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

Part II: Free Response—Show all work in the space provided

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

11. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t I smeasured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to estimate $C'(3.5)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t)=16-16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t=5$.