Name KEY Date______________________ Period________

PreAP Precalculus

TEST Chapter 3.1-3.4 Form A. No Calculator Permitted

Part I: Multiple Choice

A calculator will be permitted for this section. Put your CAPITAL LETTER answer choice in the blank to the left of the number.

* Find 2 roots from calculator, synthetically divide, then factor remaining quadratic.

Roots $x=-1, 1, 8, \frac{1}{2}$

So, $RSTV = (-1)(1)(8)\left(\frac{1}{2}\right) = -4$

E 1. Let *R, S, T,* and *V* be the roots of $2x^4 - 17x^3 + 6x^2 + 17x - 8$. Find the product *RSTV*.

(A) 24 (B) −8 (C) 8 (D) 4 (E) −4

D 2. Simplify: $i^{487} =$ $\begin{array}{r} 121 \\ 4\overline{)487} \\ R3 \end{array}$ so, $i^{487} = i^3 = -i$

(Not in 3.1-3.2) (A) i (B) -1 (C) 1 (D) $-i$ (E) 0

E 3. Which of the following MUST be true about a polynomial function of even degree?

(A) It is an even function (can, but not always) (B) It has no real solutions (can, but not always) (C) It has an odd number of complex roots (can, but not always) (Real and/or imaginary)

(D) It has at least one irrational root (can, but not always) (E) It has an odd number of relative extrema

true! up to n-1, where n is the degree

A 4. A linear factor of $x^3 - x^2 - 10x - 8$ is $(x+1)$ and what other ~~possible~~ factor?

$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$

(A) $x+2$ (B) $x+3$ (C) $x-2$ (D) $x-3$ (E) $x-1$

and $x-4$, but $x-4$ is not an answer choice.

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

B 5. The value of *k* that will make *x* + 1 a factor of $kx^3 - 17x^2 - 4kx + 8$ is:

(A) -3 (B) 3 (C) −5 (D) 4 (E) -1

$x=-1$ is a root:

@$x=-1$: $k(-1)^3 - 17(-1)^2 - 4k(-1) + 8 = 0$

$-k - 17 + 4k + 8 = 0$

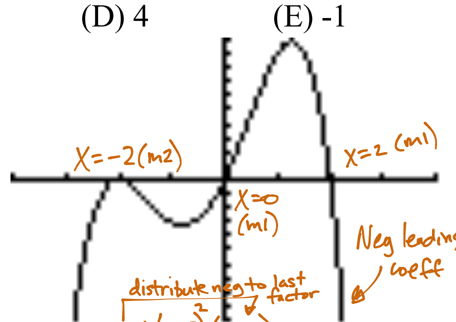
$3k = 9$, $k = 3$

C 6. Which of the specified functions might have the given graph?

(A) $f(x) = x(x+2)^2(x-2)$ (B) $f(x) = -x^2(x+2)(2-x)$

(C) $f(x) = x(x+2)^2(2-x)$ (D) $f(x) = x(x+2)(x-2)^2$

(E) $f(x) = -x(x+2)(x-2)^2$



$y = -x(x+2)^2(x-2)$

$y = x(x+2)^2(2-x)$

B 7. If a function of degree 6 has roots of –1, 2, $\sqrt{2}+1$ and $i+1$ (not on 3.1-3.2), the other root must be:

$-\sqrt{2}+1$ or $1-\sqrt{2}$ irrational conjugate

$-i+1$ or $1-i$ (complex conjugate)

(A) $\sqrt{2}-1$ (B) $1-\sqrt{2}$ (C) $i-1$ (D) −2 (E) $-i-1$

A 8. What is the remainder when $f(x) = 2(x-1)^3 + 9$ is divided by *x*? X is a factor → $x=0$ is a root

(A) 7 (B) 5 (C) 3 (D) 2 (E) 1

$f(0) = \text{Remainder} = 2(0-1)^3 + 9 = -2 + 9 = 7$

D 9. An equation of a polynomial of the form $y = Af(x)$ of lowest degree with the following characteristics $f(0) = -5$, $f(1) = 0$, $f(i) = 0$, and $f(\sqrt{2}) = 0$ has a vertical dilation value of $A =$

not on 3.1-3.2 $f(-i) = 0$ (conjugate)

$f(-\sqrt{2}) = 0$

(A) -1 (B) $\frac{2}{3}$ (C) 3 (D) $-\frac{5}{2}$ (E) $-\frac{4}{3}$

$y = (x-1)(x-i)(x+i)(x-\sqrt{2})(x+\sqrt{2})$

$y = (x-1)(x^2+1)(x^2-2)$

$-5 = (-1)(1)(-2)A$

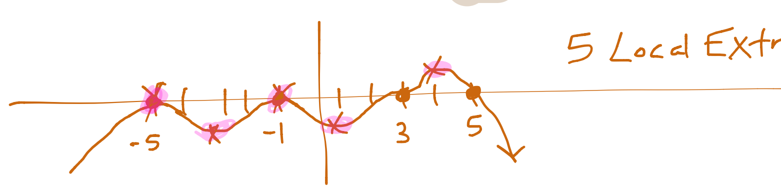
$-5 = 2A$

$A = -\frac{5}{2}$

C 10. An equation of an 8th degree polynomial with a negative leading coefficient whose only roots are $x = -5(m2)$, $x = -1(m2)$, $x = 3(m3)$ and $x = 5(m1)$ has how many relative extrema?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

5 Local Extrema



Part II: Free Response

Show all work and proper notation in the space provided below or to the right of each problem. Be sure to label your work corresponding to each part a), b), c), etc.

11. For $h(x) = -14x^6 + 166x^2 - 16x^4 + 2x^7 + 24 + 8x^5 + 30x^3 + 120x$

a) Find the range of $h(x)$

b) What is the coordinate of the y-intercept of $h(x)$?

c) List the distinct, possible rational roots

d) Given that $x = 2i$, $x = 1+\sqrt{2}$, and $x = -1$ are all roots of $h(x)$, use (and show) synthetic division to find all the exact values of the other complex roots. List all you final roots at the ends as $x =$

$h(x) = 2x^7 - 14x^6 + 8x^5 - 16x^4 + 30x^3 + 166x^2 + 120x + 24$

(a) Since $h(x)$ is an odd-degree polynomial with opposite end behaviors,

$R_f: \mathbb{R}$

(b) $h(0) = 24$, so y-int is at

$(0, 24)$

(c) $24: \pm 1, \pm 24, \pm 2, \pm 12, \pm 3, \pm 8, \pm 4, \pm 6$

$2: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 24, \pm 2, \pm 12, \pm 3, \pm 8, \pm 4, \pm 6, \pm \frac{1}{2}, \pm \cancelto{12}{\frac{24}{2}}, \pm \cancelto{1}{\frac{2}{2}}, \pm \cancelto{6}{\frac{12}{2}}, \pm \frac{3}{2}, \pm \cancelto{4}{\frac{8}{2}}, \pm \cancelto{2}{\frac{4}{2}}, \pm \cancelto{3}{\frac{6}{2}}$

So, the distinct, possible, rational roots are:

$\pm 1, \pm 24, \pm 2, \pm 12, \pm 3, \pm 8, \pm 4, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

(d) $x = 2i, -2i, 1+\sqrt{2}, 1-\sqrt{2}, x = -1$

conjugates   conjugates

	2	−14	8	−16	30	166	120	24
−1	↓	−2	16	−24	40	−70	−96	−24
	2	−16	24	−40	70	96	24	$\boxed{0}$
$2i$	↓	$4i$	$-8-32i$	$32i+64$	$-64+48i$	$12i-96$	−24	
	2	$4i-16$	$16-32i$	$32i+24$	$6+48i$	$12i$	$\boxed{0}$	
$-2i$	↓	$-4i$	$32i$	$-32i$	$-48i$	$-12i$		
	2	−16	16	24	6	$\boxed{0}$		
$1+\sqrt{2}$	↓	$2+2\sqrt{2}$	$-10-12\sqrt{2}$	$-18-6\sqrt{2}$	−6			
	2	$-14+2\sqrt{2}$	$6-12\sqrt{2}$	$6-6\sqrt{2}$	$\boxed{0}$			
$1-\sqrt{2}$	↓	$2-2\sqrt{2}$	$-12+12\sqrt{2}$	$-6+6\sqrt{2}$				
	2	−12	−6	$\boxed{0}$				

$(1+\sqrt{2})(-14+2\sqrt{2})$
$-14+2\sqrt{2}-14\sqrt{2}+4$
$-10-12\sqrt{2}$

$(1+\sqrt{2})(6-12\sqrt{2})$
$6-12\sqrt{2}+6\sqrt{2}-24$
$-18-6\sqrt{2}$

$(1+\sqrt{2})(6-6\sqrt{2})$
$6-6\sqrt{2}+6\sqrt{2}-12$
-6

$2x^2 - 12x - 6 = 0$

$2(x^2 - 6x - 1) = 0$

by Quadratic Formula with $a = 1$, $b = -6$, $c = -1$

$x = \frac{6 \pm \sqrt{36 - 4(-1)}}{2}$

$x = \frac{6 \pm 2\sqrt{10}}{2} = 3 \pm \sqrt{10}$

So, $x = -1, \pm 2i, 1 \pm \sqrt{2}, 3 \pm \sqrt{10}$