Chapter 1.2: Factoring with Expressions & Equations

Factoring is a very important, amazingly fun skill, especially for polynomials, because it "undoes" an expanded expression, with possibly many terms, into factors. It is a super-duper important skill to possess in this class. You, too, can become a fluent factored(erererer) with enough practice, practice, practice (repeat).

I. Factoring Expressions

With an expression factored, one can use the zero-product property to find the zeros of that factored expression. We'll get to solving equations soon enough. First, though, we must work with mathemtical/algebraic expressions (as opposed to colloquial expressions, such as "***Blazing Bananas, I'm Bursting with Bataquatulence***!)

Mathematical (Algebraic) Expression--An algebraic expression is a collection of either numbers and/or variables combined with mathematical operators such as multiplication, division, addition, and subtraction symbols.

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- $2x$
- $3x^4-1$
- $\frac{4\sin(x)+1}{2\sqrt{x}}+17(x-1)\ln x-9x^2e^{-4x}+\frac{®}{4}$
- $x^2-7x+10$

Terms in an expression are separated by plus or minus signs. Terms may contain several factors.

$$\underbrace{\frac{4\sin(x)+1}{2\sqrt{x}}}_{\text{term 1}}+\underbrace{17(x-1)\ln x}_{\text{term 2}}-\underbrace{9x^2e^{-4x}}_{\text{term 3}}+\frac{®}{\underbrace{4}_{\text{term 4}}}$$

Factors are separated by multiplication or division symbols. Factors may consist of terms.

$$\underbrace{\frac{4\sin(x)+1}{2\sqrt{x}}}_{\text{3 factors}}+\underbrace{17(x-1)\ln x}_{\text{3 factors}}-\underbrace{9x^2e^{-4x}}_{\text{3 factors}}+\frac{®}{\underbrace{4}_{\text{1 factor}}}$$

Example 1:

Factor each of the following expressions completely into linear or irreducible prime factors.

(a) $x^2 - 7x + 10$

(b) $x^2 + 16$

(c) $24x - 4x^3 - 4x^2$

(d) $u^3v - uv^3$

(e) $18x^3 + 12x^2 + 2x$

(f) $8x^2 - 6 - 8x$

(g) $35x^2 - x - 12$

(h) $3x^3 + x^2 - 6x - 2$

(i) $x^2(5x-2) + (3x^2-1)(10x-4)$

II. Solving Equations

Algebraic Equation is an mathematical sentence that equates two expressions. There are several types of mathematical equations. A …

- **Name Equation** is an equation that names an expression
  - $y = 2x + 4$
  - $f(x) = 2\sin(3x - 1) + 5$

- **Contradiction** is an equation that has no solutions and is never true.
  - $0 = 1$
  - $4 = 4!$
  - $\ln(1 - x) = \ln(x - 2)$

- **Identity** is an equation that is true for all values in the domain of the expression, has infinitely many solutions, and is always true.
  - $0! = 1!$
  - $\sin^2 x + \cos^2 x = 1$
  - $|x| = \sqrt{x^2}$

- **Conditional Equation** is an equation that is true for a finite number of values, or is true only in certain conditions, that is. <u>THESE ARE THE EQUATIONS WE ARE INTERESTED IN SOLVING!!!</u>

To solve a conditional equation systematically,

I. Is there only a single variable to the same power? If so, collect variables on the left side, numbers on the right (expanding if necessary), then extract the roots.

Example 2:
Solve for the indicated variable:

(a) $2(3 - 4z) - 5(2z + 3) = z - 17$

(b) $8(y^2 - 3) + 4(2y^2 - 6) = 16$

You can often “clear out” pesky fractions by multiplying through the equation by the LCM of all denominators in the equation. If you really like fractions, or like to build your mental skills, you may work them WITH the fractions.

Example 3:
Solve for the indicated variable.

(a) Keep Fractions

$$\frac{t-1}{3}+\frac{t+5}{4}=\frac{1}{2}$$

(b) Clear out Fractions

$$\frac{t-1}{3}+\frac{t+5}{4}=\frac{1}{2}$$

II. Are there two or more powers of a single variable? Use the <u>Zero Product Property</u>.

Zero Product Property

If $a \cdot b = 0$, then either $a = 0$ or $b = 0$ (or both).

This means the product of factors equals zero if and only if at least one of the factors is zero, and it ONLY works for zero!

Our goal, typically, is to factor a polynomial into its “prime” linear factors of the form $ax+b$, $a,b \in \mathbb{R}$, $a \neq 0$. So how can we systematically reach our goal to solve equations?

THE FIRST RULE OF FACTORING IS TO LOOK FOR A COMMON FACTOR (GCF)!!!

NEVER, NEVER, NEVER divide out a variable from a conditional equation. WHEN YOU LOSE VARIABLE INFORMATION, YOU LOSE POSSIBLE SOLUTIONS.

For quadratics (an polynomial equations in general), we must put the equation in standard form (descending order): $ax^2+bx+c=0$

Here we go!

Example 4:
Solve for x.

(a) $x^2+25=10x$ (b) $2x^2+10x=48$ (c) $(x+2)(x+3)=12$ (d) $3x^2=4x$

(e) $3x^2+18x=6x^3$ (f) $2x(x+1)=7x-2$ (g) $x^3+3x=5x^2+15$ (h) $3x^4+6x^3-27x^2=54x$

Example 5:
Verify example 4(h) using the graphing calculator.