Chapter 5.2: Applications of Angles

From the formal definition of a radian measure of an angle comes a handy formula which allows us to find the length of an arc along a circle.

Definition

If θ is a central angle in a circle of radius *r*, where θ is measured in radians, then the **arc length** *s* of the intercepted arc is given by

$$s = r\theta$$

Example 1:
Find the distance a runner travels (in miles) along a circular track of radius 5 miles if he runs through an angle of 300°

Example 2:
If the same runner on the same track from Example 1 runs an ultra-marathon of 150 miles around that same track, through how many degrees did he run?

Once we take a slice of a circle, we can determine its area.

Example 3:
Derive the formula for the area of a sector of a circle with a radius *r* and a central angle of θ.

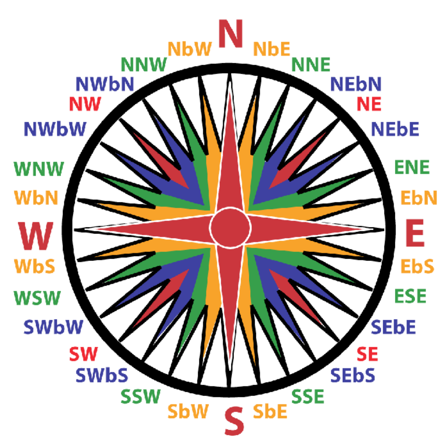
Definition

If θ is a central angle in a circle of radius *r*, where θ is measured in radians, then the area of the sector is given by

$$A = \frac{1}{2} r^2 \theta$$

Example 4:
Mr. Kilford just scarfed down a slice of Freudian Pizza that had a 40° central angle (he always measures them before he eats them just to make me happy). If the slice came from a 14-inch diameter Freudian Pizza (a) find the perimeter of the slice Kilford just ate (b) determine the square **footage** of pizza Kilford consumed in that single slice of Freudian Pizza. (Hint: Someone whose actual foot is actually one foot long has a foot that is actually 1/5280 mile long.)

In navigation, the heading, bearing, or course of an object is usually given as a line of travel measured **clockwise** from **due north**. For instance, a plane flying on a heading of 270° will be flying due west. A boat on a bearing of 135° will be boating in a SE direction.



Example 5:
Find the angle measure in decimal degrees and decimal radians that describes the following bearings /headings/courses.
(a) SW (b) NNW (c) SEbE

There's one last thing we need to cover in this comprehensive, awesome section. Often, situations in which we measure angles have angles that change. In these cases, we want to measure how fast these angles are changing. This is called a**ngular velocity**. We typically measure this in radians per unit of time, degrees per unit of time, or revolutions per unit of time.

Definitions

The **angular velocity** ω (omega) of a point moving in a circular path is the angle through which it travels θ per unit of time *t*. This is given by

$$\omega = \frac{\theta}{t}$$

If an object moves along a circle of radius *r* at a constant rate, its **linear velocity** *V* is the distance *s* traveled along the circumference of the circle per unit of time *t*. Thus,

$$V = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

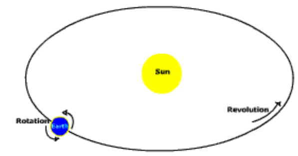
Example 6:
Determine the angular velocity, in radians per second, and the linear velocity in inches per second of the tip of the second hand of a clock that is 5 inches long.

Example 7:
Mr. Korpi is taping ants down to a 12-inch diameter LP (long play) vinyl record to spin on his record player. If he plays the record at the recommended 78 rpm, what is the liner velocity, in feet per minute, of the taped ants taped down at the following distances from the center of the record?

(a) 1 inch (b) 3 inches (c) 6 inches

Example 8:
Mrs. Lincoln's vehicle has wheels that are 36 inches in diameter. If she is cruising through Landa Park in her vehicle at 20 mph, how many revolutions per **minute** are her wheels making?

Example 9:
Hold on to your homework! (a) Approximate the linear speed of the earth in its orbit about the sun in mph. Assume that the orbit of the earth about the sun is nearly a circle with a radius of 93,000,000 miles. (b) If the radius of the earth is about 3960 miles, what is the linear velocity of a math student around the earth's axis if he is vacationing in Ecuador?



Example 10:
A bicycle has a pedal sprocket with a 9 in diameter, a wheel sprocket with a 3 inch diameter, and a wheel with a 29 inch diameter. If the bike is being pedaled at 35 RPMs, how fast, in mph, is the bike travelling down the road?