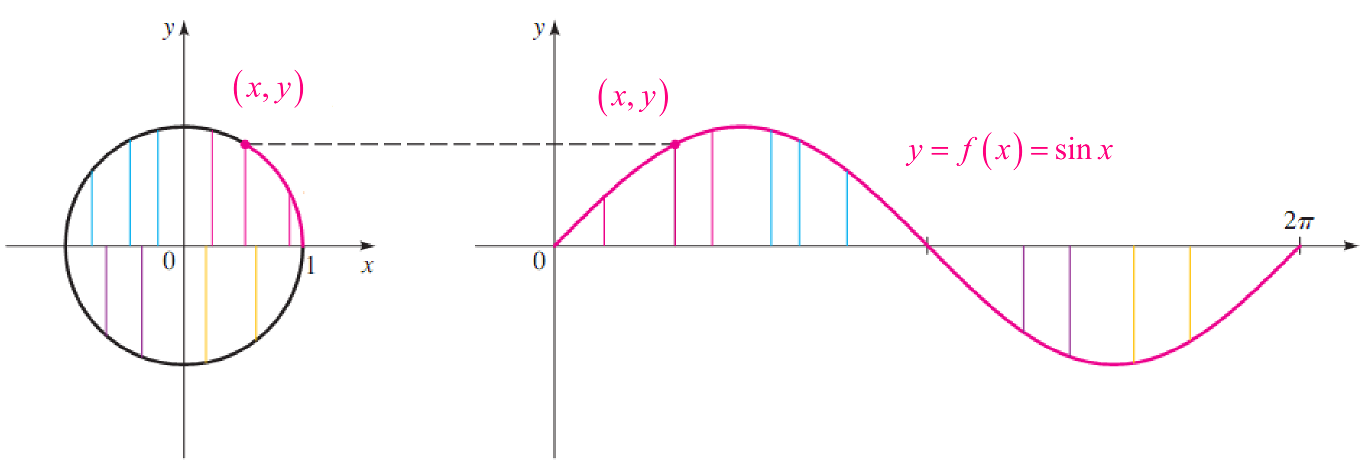
Chapter 5.4: Sinusoids

If we take our circular functions and "unwrap" them, we can begin to look at the graphs of each trig function's ratios as a function of the angle in radians.

We will begin by looking at two trig functions that are very useful and share many common traits: sine and cosine.

Example 1:
Make a table of values for θ and $\sin\theta$ for values of θ between 0 and 2π. Start by choosing quadrantal values of θ, then fill in with other unit circle angles as necessary. Converting all values to decimals will help. Plot each of your ordered pairs on the Cartesian plane. Connect the dots. What do you notice?

Notice that the graph not only passes the vertical line test (so it IS a function), but it seems to resemble a "wave" pattern, repeating itself every 2π radians. For this reason, we call the function $f(\theta)=\sin\theta$ a periodic function, with a period, P, of 2π. Because we prefer to use x as our independent variable for function, we will forevermore be referring to this function as $f(x)=\sin x$, where x resembles and angle in radians and $f(x)$ represents the vertical to hypotenuse ratio for the unique reference triangle formed by the independent angle x.

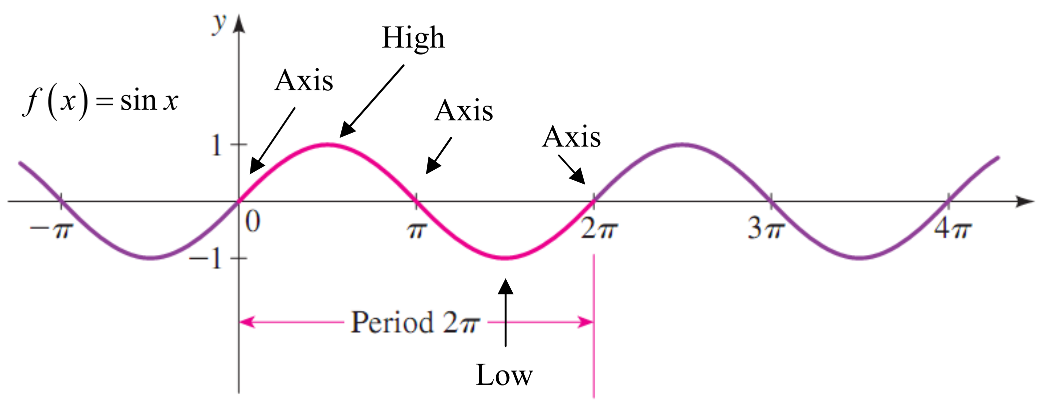


This graph of $f(x)=\sin x$ is a new parent function. It is a periodic functions that repeats itself every 2π radians. Each length of 2π represents a new **cycle** of the graph. Because of the shape and smooth continuity of this wave function, it belongs to the class of periodic functions called **sinusoids**.

We need to learn as much as we can about it and be able to sketch it quickly and accurately so that we can eventually graph transformations of it. Notice that it takes 5 critical values to accurately sketch one cycle of $f(x)=\sin x$ from 0 to 2π. These are the quadrantal angles. Any easy way to remember this is to remember . . .

SAHALA

S Sine graph
A Axis
H High
A Axis
L Low
A Axis



Sinusoidal wave functions have their own unique characteristics and nomenclature. In order to define these new terms, it's easier to first look at the standard transformation form of our new parent function $f(x)=\sin x$.

Standard Transformation form of $f(x)=\sin x$

The standard transformation form of $f(x)=\sin x$ is given by

$$f(x)=A\sin\left(B(x-C)\right)+D$$

where
$|A|$ is the **amplitude** of the wave
$|B|$ is the **number of cycles** in 2π
C is the **phase shift**
D is the vertical location of the **sinusoidal axis**
P is the **period** of the function or the length of one cycle. Also known as **wavelength**. $P=\dfrac{2\pi}{|B|}$

Note: frequency, another way to measure cycle length, is the reciprocal of the period.

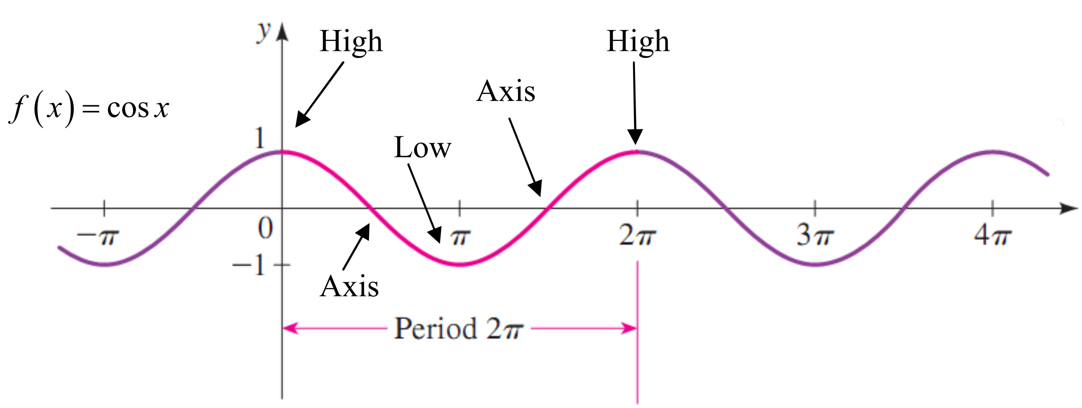
Example 2:
Sketch one cycle of $f(x) = \sin x$ using the SAHALA method. List as many characteristics as you can about the function $f(x) = \sin x$ including domain, range, symmetry, end behavior, intercepts, maximum and minimum values, curvature changes, etc. List all the information from the previous chart as well.

Example 3:
Make a table of values for θ and $\cos\theta$ for values of θ between 0 and 2π. Start by choosing quadrantal values of θ, then fill in with other unit circle angles as necessary. Converting all values to decimals will help. Plot each of your ordered pairs on the Cartesian plane. Connect the dots. What do you notice?

The easy way to remember how to sketch one cycle of the graph of $f(x) = \cos x$ is **CHALAH**

CHALAH

C Cosine graph
H High
A Axis
L Low
A Axis
H High



Example 4:
Sketch one cycle of $f(x)=\cos x$ using the CHALAH method. List as many characteristics as you can about the function $f(x)=\cos x$ including domain, range, symmetry, end behavior, intercepts, maximum and minimum values, curvature changes, etc. List all the information from the previous chart as well. Also, describe any similarities and differences between the graphs of $f(x)=\cos x$ and $f(x)=\sin x$.

Because sine and cosine are so similar, only differing by a 90° phase shift, we call them **complementary functions**. This is actually what the "co" means in "cosine." Additionally, sine and cosine, both containing the word "sine," make up the two functions also known as sinusoids.

Now we can sketch transformations of our sinusoids.

Example 5:
Sketch three positive cycles of $f(x)=2\cos 3x$. Find the new period, then find and label the new critical values needed for each cycle.

When sinusoids have a phase and/or vertical shift, it is helpful to identify where the old *x*- and *y*- axis have moved to, especially when you are using the SAHALA and/or CHALAH method. It is always important to draw your graph intersecting the actual *y*-axis.

Example 6:
Sketch at least one cycle of $f(x)=3\sin\left(x-\frac{\pi}{2}\right)-2$. Find and show the new sinusoidal axis and the "new" *y*-axis.

Sometimes the value of A in standard transformation form is negative. In this case, the graph will reflect across the x-axis (the original sinusoidal axis). To easily sketch such a graph, identify the new sinusoidal axis, then switch your "high" and "low" points. The "axis" points will stay the same. If you like, you can think of it in terms of the following:

-SALAHA and/or -CLAHAL

We can now include one of each of the transformations.

Example 7:

Sketch at least one cycle of $f\left(x\right)=1-\frac{1}{2}\cos\left(4x+\pi\right)$. Label the critical values and the new sinusoidal axis. Once you have the graph, write an equivalent equation in terms of (a) positive cosine (b) sine and (c) negative sine.

Example 8:

Sketch at least one cycle of $f\left(x\right)=3-2\sin\left(2x-\frac{\pi}{2}\right)$. Label the critical values and the new sinusoidal axis. Determine how you can find the range of the function prior to graphing. Once you have the graph, write an equivalent equation in terms of (a) positive sine (b) positive cosine, and (c) negative cosine.

As you discovered above, the range of a sinusoid in standard transformation form $f(x) = A\sin(B(x-C)) + D$ is given by $D \pm |A|$.

Example 9:

Sketch at least one cycle of $f(x) = -2\sin\left(-\pi x + \frac{\pi}{2}\right) - 5$. Label the critical values and the new sinusoidal axis. Determine the range of the function prior to graphing. Use the fact that sine is an odd function to help you simplify the equation prior to graphing. Once you have the graph, write an equivalent equation of the graph in terms of positive and negative cosine.

Example 10:

Sketch at least one cycle of $f(x) = 2 - 4\cos(-\pi - 3x)$. Label the critical values and the new sinusoidal axis. Determine the range of the function prior to graphing. Use the fact that cosine is an even function to help you simplify the equation prior to graphing. Once you have the graph, write an equivalent equation of the graph in terms of positive and negative sine.

Sometimes the new period and the phase shift lead to critical values that aren't so "nice" or easily found. In such a case, it is wise to use a "sacrificial" x-axis off to the side to get the values and spacing of the final critical values.

Example 11:

Sketch at least one cycle of $f(x) = 3 + 2\cos\left(\frac{\pi x}{5} - \frac{3\pi}{5}\right)$. Label the critical values and the new sinusoidal axis. Determine the range of the function prior to graphing. Once you have the graph, write an equivalent equation of the graph in terms of positive and negative sine and negative cosine. YOUR FINAL GRAPH SHOULD ONLY SHOW THE ACTUAL, FINAL CRITICAL x-VALUES!

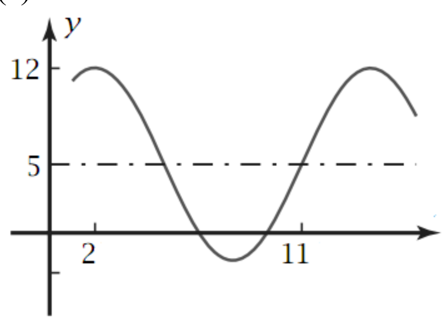
Example 12:

Sketch at least one cycle of $f(x) = -\frac{2}{3}\sin\left(\frac{2x}{3} + \frac{\pi}{3}\right) - 1$. Label the critical values and the new sinusoidal axis. Determine the range of the function prior to graphing. Once you have the graph, write an equivalent equation of the graph in terms of positive and negative cosine and positive sine. YOUR FINAL GRAPH SHOULD ONLY SHOW THE ACTUAL, FINAL CRITICAL x-VALUES!

Example 13:

For each of the following, write a sinusoidal equation in terms of both sine and cosine.

(a)



(b)