Chapter 6.3: Composite Identities

Have you ever wanted to do something, but you didn't think it was right?

Perhaps you wanted to say that $\ln(a+b)$ was equal to $\ln a+\ln b$. Of course it is not equal.

If you were asked to "expand" the expression $\cos(x+y)$, would you be tempted to say it was $\cos x+\cos y$? I hope your answer is "No!"

There is a way to do it, but not by "distributing" the cosine. The angle $x+y$ is called a **composite angle**, because it combines two angles x and y to form a new angle.

Composite Identities for Cosine

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$
$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

Notice the sign change. We can summarize both as

$$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$$

Example 1:
Find the simplified, exact value of $\cos 15^\circ$ by finding two angles from the Unit Circle that either add or subtract to give 15°. Verify on your calculator.

Exact values of trig ratios of any angle that is a multiple of 15° or $\frac{\pi}{12}$ radians can be found using a composite of Unit Circle angles.

Example 2:

Find the simplified, exact value of $\cos\frac{7\pi}{12}$ by finding two angles from the Unit Circle that either add or subtract to give $\frac{7\pi}{12}$. Verify on your calculator.

Example 3:

Using the cofunction identity $\sin x = \cos\left(\frac{\pi}{2} - x\right)$, derive an identity for $\sin(x+y)$.

Composite Identities for Sine

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$
$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

Notice the sign does NOT change. We can summarize both as

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

Example 4:
Use the composite identities to prove the following cofunction identity:

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x$$

Example 5:
Find the simplified, exact value of $\sin\frac{35\pi}{12}$ by finding two angles from the Unit Circle that either add or subtract to give $\frac{35\pi}{12}$ (or an angle coterminal with it). Verify on your calculator.

Example 6:
Write each of the following as the sine or cosine of a single angle:

(a) $\sin 22^\circ \cos 13^\circ + \cos 22^\circ \sin 13^\circ$

(b) $\sin x \sin 2x - \cos x \cos 2x$

Composite Identities for Tangent

$$\tan(x \pm y) = \frac{\sin(x \pm y)}{\cos(x \pm y)} \quad \text{or} \quad \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Example 7:

Find the simplified, rationalized, exact value of $\tan \frac{5\pi}{12}$ using BOTH of the identities above. Verify on your calculator.

Example 8:
Prove the following identities:

(a) $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1 + \tan\theta}{1 - \tan\theta}$

(b) $\sin(x - y) + \sin(x + y) = 2\sin x \cos y$

Example 9:
Prove the following identity:

$$\sin 3u = 3\cos^2 u \sin u - \sin^3 u$$