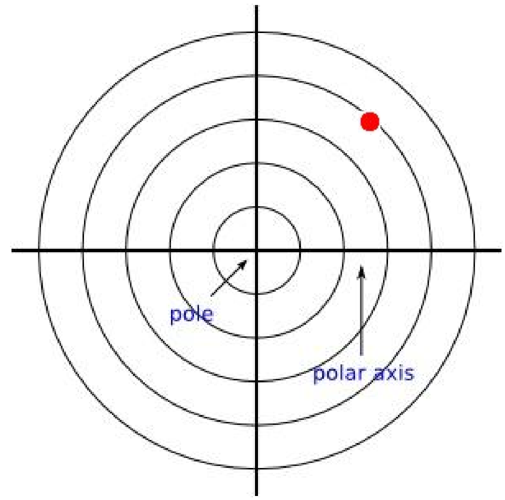
Chapter 7.1: Polar Coordinates

Up to now, we have been navigating through 2-D Euclidean space by using a Cartesian, or **rectangular coordinate system**. Each point in Euclidean space can be expressed by a rectangular coordinate (x, y). With our new developed skill of using the unit circle, we are now ready to explore a different way to navigate around Euclidean space, using a **polar coordinate system**.



A polar coordinate is an ordered pair (r, θ), where θ represents an angle in standard position with its tail at the **pole** (origin) and head out along the **polar axis** (positive *x*-axis), and *r* represents a radius of rotation (or distance from the pole).

For graphs that pass the vertical line test, rectangular coordinates are ideal for expressing *y*-values as a function of *x*, $y = f(x)$, but for more elaborate curves, such as circles, limaçons, cardioids, rose curves, and lemniscates (you'll meet these guys later), are better represented as polar equations, expressing radii as functions of an angle θ, $r = f(\theta)$.

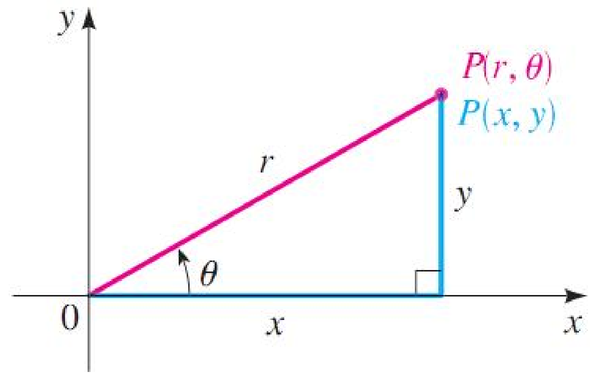
Important Note
In a rectangular coordinate, we list the independent variable, *x*, first and the dependent variable, *y*, second, but in a polar coordinate, we list the independent variable, θ, second and the dependent variable, *r,* first.

Example 1:
Plot the following polar coordinates, then list three other, equivalent polar coordinates such that $-2\pi \le \theta \le 2\pi$.

(a) $\left(2, \frac{3\pi}{4}\right)$ (b) $\left(4, -\frac{\pi}{6}\right)$ (c) $\left(\frac{3}{2}, -\frac{3\pi}{2}\right)$ (d) $\left(-3, \frac{5\pi}{3}\right)$ (e) $(-\pi, 0)$

For any point in 2-D space, there is **only one rectangular coordinate** associated with it, but that same point may be expressed equivalently by **infinitely many different polar coordinates**.

There will times where we are interested in converting between rectangular and polar coordinates. The geometric tools of right triangles are all we need to do so, namely the Pythagorean Theorem and sine, cosine, and tangent.



Relationship between Polar and Rectangular Coordinates

Rectangular coordinates are in the form (x, y), where x is the independent variable.

Polar coordinates are in the form (r, θ), where θ is the independent variable.

For coordinate conversions:

Polar to Rectangular	**Rectangular to Polar**
$x = r\cos\theta$	$r^2 = x^2 + y^2$
$y = r\sin\theta$	$\tan\theta = \dfrac{y}{x},\ x \neq 0$

Example 2:
Find the rectangular coordinate for the point whose polar coordinates are

(a) $\left(-5, -\dfrac{4\pi}{3}\right)$

(b) $\left(4, -\dfrac{5\pi}{6}\right)$

Example 3:
Convert the following rectangular coordinate into four different, equivalent polar coordinates.

(a) $(-2, 2)$

(b) $(-5, -8)$, calculator permitted

Important Note
The equations we use to find equivalent polar coordinates from existing rectangular coordinates do not uniquely determine r or θ. It is OUR job to make sure that the values we ultimately chose for r and θ give us the point we want in its correct quadrant.

We will now perform the same translations between rectangular and polar for equations. It requires nothing more than the same right triangle tools from the box above.

Example 4:
Express the following rectangular equations in terms of polar equations, solve each polar equation for the dependent variable r, if possible, so we can graph it. Sketch your result on your calculator in polar, radian mode. . Explore for different values of θ_{min}, θ_{max}, and θ_{step}.

(a) $x^2 = 5y$ (b) $y = 2x + 1$ (c) $y = 4$ (d) $x^2 + y^2 = 16$

When converting from polar to rectangular, it is easier to replace any existing trig function with its definition in terms of x, y, and r. After this, your r values will usually either divide out or become r^2. Deal with it separately in a second step. Solve for y, if possible.

Example 5:
Express the following polar equations in terms of rectangular equations. If you can, determine the graph of the equation from its rectangular equation:

(a) $r = 5\sec\theta$ (b) $r = 3\sin\theta$ (c) $r = 3 - 3\cos\theta$ (d) $\theta = \dfrac{3\pi}{4}$