Part 1

1) $(\sin A + \cos A)^2 \stackrel{?}{=} 1 + 2\cos A \sin A$

$\sin^2 A + 2\cos A \sin A + \cos^2 A$

$1 + 2\cos A \sin A$

— I ♡ PIDs!

2) $\sec x(\sec x + 1) = \dfrac{\tan^2 x}{1 - \cos x}$

$\dfrac{\sec^2 x - 1}{1 - \frac{1}{\sec x}} \left(\dfrac{\sec x}{\sec x}\right)$

$\dfrac{\sec x(\sec^2 x - 1)}{\sec x - 1}$

$\dfrac{\sec x(\sec x + 1)(\sec x - 1)}{(\sec x - 1)}$

$\sec x(\sec x + 1)$

— Done

3) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

Ada)

$\dfrac{\sec x \sin x + \cos x \csc x}{\csc x \sec x}$

$\dfrac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{1}{\cos x \sin x}}$

$\dfrac{1}{\cos x \sin x} \Big/ \dfrac{\cos x \sin x}{\cos x \sin x}$

$\sin^2 x + \cos^2 x$

— Cool Beans!

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4) $\frac{\sec^2 x - 7\tan x + 11}{\sec^2 x - 17} = \frac{\tan x - 3}{\tan x + 4} \cdot \frac{(\tan x - 4)}{(\tan x - 4)}$

$\frac{\tan^2 x - 7\tan x + 12}{\tan^2 x - 16}$

$\frac{(\sec^2 x - 1) - 7\tan x + 12}{\sec^2 x - 1 - 16}$

$\frac{\sec^2 x - 7\tan x + 11}{\sec^2 x - 17}$ — QED

5) $\frac{1 - \cos B}{\sin B} = \frac{\sin B}{1 + \cos B} \cdot \frac{(1 - \cos B)}{(1 - \cos B)}$

$\frac{\sin B(1 - \cos B)}{1 - \cos^2 B}$

$\frac{\sin B(1 - \cos B)}{\sin^2 B}$

$\frac{1 - \cos B}{\sin B}$ — Done

6) $\frac{\sin A}{1 + \sec A} = \frac{\sin A \cos A}{\cos A + 1}$

$\frac{\sin A}{1 + \frac{1}{\cos A}} \cdot \frac{(\cos A)}{(\cos A)}$

$\frac{\sin A \cos A}{\cos A + 1}$ — Done

Part II

1) $\sin 2x = -2\cos x$

$2\sin x\cos x + 2\cos x = 0$

$2\cos x(\sin x + 1) = 0$

$2\cos x = 0 \qquad \sin x + 1 = 0$

$\cos x = 0 \qquad \sin x = -1$

$x = \cos^{-1}(0) \qquad x = \sin^{-1}(-1)$

$x = \frac{\pi}{2} + \pi N \qquad x = \frac{3\pi}{2} + 2\pi N$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

2) $-2\cos^2 x - 5\sin x + 4 = 0$ ← or ... This is a better move

$-2\left(\frac{1}{2}(1+\cos 2x)\right) - 5\sin x + 4 = 0$

$-1 - \cos 2x - 5\sin x + 4 = 0$

$-\cos 2x - 5\sin x + 3 = 0$

$-(1 - 2\sin^2 x) - 5\sin x + 3 = 0$

$-1 + 2\sin^2 x - 5\sin x + 3 = 0$

$2\sin^2 x - 5\sin x + 2 = 0$

$(2\sin x - 1)(\sin x - 2) = 0$

$2\sin x - 1 = 0 \qquad \sin x = 2$ ← no solution

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$-2(1-\sin^2 x) - 5\sin x + 4$

$-2 + 2\sin^2 x - 5\sin x + 4$

$2 + 2\sin^2 x - 5\sin x$

but gets to the same place

3) $5\sqrt{3}\tan x + 3 = 8\sqrt{3}\tan x$

$5\sqrt{3}\tan x - 8\sqrt{3}\tan x + 3 = 0$

$\frac{-3\sqrt{3}}{-3\sqrt{3}}\tan x = \frac{-3}{-3\sqrt{3}}$

$\tan x = \frac{1}{\sqrt{3}}$

$\tan x = \frac{\sqrt{3}}{3}$

$x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\boxed{x = \frac{\pi}{6}, \frac{7\pi}{6}}$

Part III

(1) $\sqrt{2}\cos\left(x-\frac{\pi}{4}\right) = \cos x + \sin x$

$\sqrt{2}\left(\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}\right)$

$\sqrt{2}\left(\cos x\frac{\sqrt{2}}{2}+\sin x\frac{\sqrt{2}}{2}\right)$

$\cos x\left(\frac{2}{2}\right)+\sin x\left(\frac{2}{2}\right)$

$\cos x+\sin x$ — DONE

(2) $\tan A = \frac{1-\cos 2A}{\sin 2A}$

$\frac{1-(1-2\sin^2 x)}{2\sin A\cos A}$

$\frac{1-1+2\sin^2 x}{2\sin A\cos A}$

$\frac{2\sin^2 y}{2\sin A\cos A}$

$\frac{\sin A}{\cos A}$

$\tan A$ — DONE

3) $\tan^2\frac{B}{2} = \csc^2 B - 2\cot B \csc B + \cot^2 B$

$(\csc B - \cot B)^2$

$\left(\frac{1}{\sin B} - \frac{\cos B}{\sin B}\right)^2$

$\left(\frac{1-\cos B}{\sin B}\right)^2$

$\frac{(1-\cos B)^2}{\sin^2 B}$

Done

$\frac{1-\cos(2(\frac{B}{2}))}{1+\cos(2\frac{B}{2})}$

$\frac{1-\cos B(1-\cos B)}{1+\cos B(1-\cos B)}$

$\frac{1-2\cos B+\cos^2 B}{1-\cos^2 B}$

$\frac{(1-\cos B)^2}{\sin^2 B}$

4) $\tan\left(\frac{5\pi}{4}-\theta\right) = \frac{1-\tan\theta}{1+\tan\theta}$

$\frac{\tan\frac{5\pi}{4}-\tan\theta}{1+\tan\frac{5\pi}{4}\tan\theta}$

$\frac{1-\tan\theta}{1+\tan\theta}$

Done

5) $\cos 3x = \cos^3 x - 3\sin^2 x \cos x$

$\cos(2x + x)$

$\cos 2x \cos x - \sin 2x \sin x$

$(\cos^2 x - \sin^2 x)\cos x - 2\sin x \cos x \sin x$

$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x$

$\cos^3 x - 3\sin^2 x \cos x$ — Done

6) $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$\dfrac{1 - \frac{1-\cos 2\theta}{1+\cos 2\theta}}{1 + \frac{1-\cos 2\theta}{1+\cos 2\theta}} \left(\dfrac{1+\cos 2\theta}{1+\cos 2\theta}\right)$

$\dfrac{1 + \cos 2\theta - (1 - \cos 2\theta)}{1 + \cos 2\theta + (1 - \cos 2\theta)}$

$\dfrac{1 + \cos 2\theta - 1 + \cos 2\theta}{1 + \cos 2\theta + 1 - \cos 2\theta}$

$\dfrac{2\cos 2\theta}{2}$

$\cos 2\theta$ — Done