Name_____ Date_____ Period_____

Worksheet 3.5—Rational Functions

Show all work. All answers must be given as **simplified, exact answers**! No Calculators are permitted unless specified otherwise.

Multiple Choice

1. Let $f(x) = -\dfrac{2x}{x^2 + 3x}$. For what values of x does the graph of $f(x)$ have a vertical asymptote?

      (A) $x = 0$ (B) $x = 0$, $x = 3$ (C) $x = 3$ (D) $x = -3$ (E) $x = 0$, $x = -3$

2. Let $f(x) = -\dfrac{2x^2}{x^2 + 3x - 4}$. Which of the following is an equation of an asymptote of $f(x)$?

      (A) $y = 2$ (B) $x = 1$ (C) $x = 4$ (D) $x = -2$ (E) $y = -4$

3. Let $f(x) = \dfrac{x^2}{x + 5}$. Which of the following statements is true about the graph of f?

    (A) There is no VA   (B) There is an HA but no VA   (C) There is an SA but no VA
        (D) There is a VA and an SA   (E) There is a VA and an HA

4. What is the degree of the end-behavior model of $f(x) = \dfrac{x^8 + 1}{x^4 + 1}$?

        (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

5. The equation of the end-behavior model of $f(x) = \dfrac{2x^3 - x + 6}{x + 2}$ is given by

(A) $y = 2x^2 - 7$ (B) $y = 2x^2 - 1$ (C) $y = 2x^2 + 4x + 7$ (D) $y = 2x^2 - 4x + 7$ (E) $y = 2x^2 - 4x - 7$

Short Answer

6. Find the x- and y- intercepts of the following functions

    (a) $t(x) = \dfrac{x^2 - x - 2}{x - 6}$ (b) $r(x) = \dfrac{x^3 - 9x}{x^3}$

7. Find all vertical and horizontal asymptotes (if any).

    (a) $k(x) = \dfrac{6x - 2}{x^2 + 5x - 6}$ (b) $j(x) = \dfrac{3x^2}{5 + 2x + x^2}$ (c) $careful(x) = \dfrac{2x + x^3}{x - 1}$

8. Analyze the following functions. As in the notes, find the domain, discontinuities, intercepts, and end-behavior. Sketch a graph. Find the equations of all HA's, VA's, and SA's. Give the coordinate of any hole. Find the range after you graph it.

(a) $f(x) = \dfrac{4x^2 + 4x - 24}{2x^2 + 4x - 16}$

(b) $h(x) = \dfrac{x - 3}{x^2 + 3x}$

(c) $q(x) = \dfrac{2x^3 - 6x^2 - 14x}{x^2 + 3x}$

(d) $t(x) = \dfrac{\left(x^2 - x - 2\right)(x - 3)}{x^2 - 4x + 3}$

9. Write and equation of a function, $f(x)$, with a VA at $x = -1$, a hole at $x = 3$, and x-intercept at $x = -3$, and an HA at $y = 1$. Once you have the equation, find $\lim_{x \to 3} f(x)$.

10. Write an equation of a function $d(x)$ with a y-intercept of $(0, -2)$, a VA at $x = 1$, an SA at $y = 2x + 7$, and a hole at $x = 2$, As $x \to \infty$, what do the slopes of the graph of $d(x)$ approach?

11. Analyze and sketch $h(x) = \dfrac{x^5 - 1}{x + 2}$. Show all asymptotes, including end-behavior asymptotes.

12. (Calculator permitted) A drug is administered to a patient, and the concentration of the drug in the bloodstream is monitored. At time $t \geq 0$ (in hours since giving the drug), the concentration (in mg/L) is given by

$$c(t) = \frac{5t}{t^2 + 1}$$

Graph the function with your graphing calculator in a reasonable window.

(a) What is a reasonable X and Y window? Justify.

(b) What is the highest concentration of drug that is reaching in the patient's bloodstream? How do you know this?

(c) What happens to the drug concentration after a long period of time? What are the mathematical implications of this if the person lives for many, many, many years after the injection?

(d) What is the concentration after 5 hours?

(e) How long does it take for the concentration to drop below 0.3 mg/L?