

Name _____ Date _____ Period _____

Worksheet 3.5—Rational Functions

Show all work. All answers must be given as **simplified, exact answers!** No Calculators are permitted unless specified otherwise.

Multiple Choice

1. Let $f(x) = -\frac{2x}{x^2 + 3x}$. For what values of x does the graph of $f(x)$ have a vertical asymptote?
(A) $x = 0$ (B) $x = 0, x = 3$ (C) $x = 3$ (D) $x = -3$ (E) $x = 0, x = -3$

2. Let $f(x) = -\frac{2x^2}{x^2 + 3x - 4}$. Which of the following is an equation of an asymptote of $f(x)$?
(A) $y = 2$ (B) $x = 1$ (C) $x = 4$ (D) $x = -2$ (E) $y = -4$

3. Let $f(x) = \frac{x^2}{x+5}$. Which of the following statements is true about the graph of f ?
(A) There is no VA (B) There is an HA but no VA (C) There is an SA but no VA
(D) There is a VA and an SA (E) There is a VA and an HA

4. What is the degree of the end-behavior model of $f(x) = \frac{x^8 + 1}{x^4 + 1}$?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4
5. The equation of the end-behavior model of $f(x) = \frac{2x^3 - x + 6}{x + 2}$ is given by
(A) $y = 2x^2 - 7$ (B) $y = 2x^2 - 1$ (C) $y = 2x^2 + 4x + 7$ (D) $y = 2x^2 - 4x + 7$ (E) $y = 2x^2 - 4x - 7$

Short Answer

6. Find the x - and y - intercepts of the following functions

(a) $t(x) = \frac{x^2 - x - 2}{x - 6}$

(b) $r(x) = \frac{x^3 - 9x}{x^3}$

7. Find all vertical and horizontal asymptotes (if any).

(a) $k(x) = \frac{6x - 2}{x^2 + 5x - 6}$

(b) $j(x) = \frac{3x^2}{5 + 2x + x^2}$

(c) $careful(x) = \frac{2x + x^3}{x - 1}$

8. Analyze the following functions. As in the notes, find the domain, discontinuities, intercepts, and end-behavior. Sketch a graph. Find the equations of all HA's, VA's, and SA's. Give the coordinate of any hole. Find the range after you graph it.

(a) $f(x) = \frac{4x^2 + 4x - 24}{2x^2 + 4x - 16}$

(b) $h(x) = \frac{x - 3}{x^2 + 3x}$

(c) $q(x) = \frac{2x^3 - 6x^2 - 14x}{x^2 + 3x}$

(d) $t(x) = \frac{(x^2 - x - 2)(x - 3)}{x^2 - 4x + 3}$

9. Write an equation of a function, $f(x)$, with a VA at $x = -1$, a hole at $x = 3$, and x -intercept at $x = -3$, and an HA at $y = 1$. Once you have the equation, find $\lim_{x \rightarrow 3} f(x)$.
10. Write an equation of a function $d(x)$ with a y -intercept of $(0, -2)$, a VA at $x = 1$, an SA at $y = 2x + 7$, and a hole at $x = 2$. As $x \rightarrow \infty$, what do the slopes of the graph of $d(x)$ approach?
11. Analyze and sketch $h(x) = \frac{x^5 - 1}{x + 2}$. Show all asymptotes, including end-behavior asymptotes.

12. (Calculator permitted) A drug is administered to a patient, and the concentration of the drug in the bloodstream is monitored. At time $t \geq 0$ (in hours since giving the drug), the concentration (in mg/L) is given by

$$c(t) = \frac{5t}{t^2 + 1}$$

Graph the function with your graphing calculator in a reasonable window.

- (a) What is a reasonable X and Y window? Justify.
- (b) What is the highest concentration of drug that is reaching in the patient's bloodstream? How do you know this?
- (c) What happens to the drug concentration after a long period of time? What are the mathematical implications of this if the person lives for many, many, many years after the injection?
- (d) What is the concentration after 5 hours?
- (e) How long does it take for the concentration to drop below 0.3 mg/L?