Name____ Date_______________________ Period______

Worksheet 5.5—Application of Sinusoids

Show all work. A calculator **is permitted**. Report <u>three decimals and units</u> in all final answers.

Multiple Choice

1. The height, h, in meters, above the ground of a car as a Ferris Wheel rotates can be modeled by the function $h(t)=16\cos\left(\frac{\pi t}{120}\right)+18$, where t is the time, in seconds.

   I. What is the radius of the Ferris Wheel?

   (A) 18 m (B) 8 m (C) 16 m (D) 9 m (E) 2 m

   II. What is the maximum height of the car?

   (A) 18 m (B) 26 m (C) 16 m (D) 17 m (E) 34 m

   III. How long does it take for the wheel to make one revolution?

   (A) 120 s (B) $\frac{\pi}{120}$ s (C) 240 s (D) $\frac{120}{\pi}$ s (E) 60 s

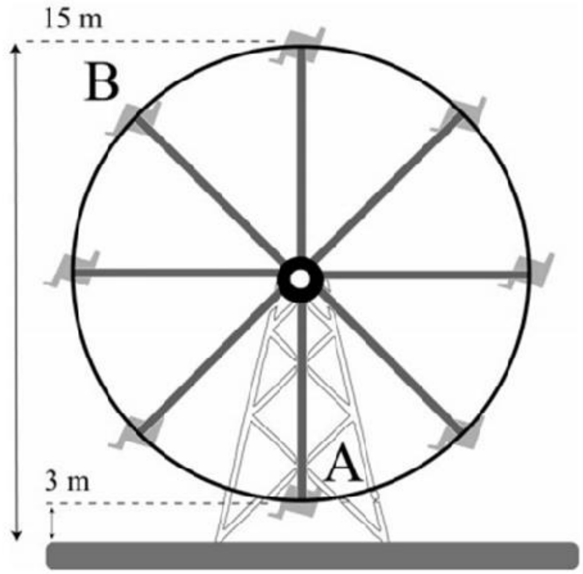
   IV. What is the minimum height of the car?

   (A) 2 m (B) 4 m (C) 16 m (D) 18 m (E) 0 m

   V. How fast does the wheel rotate?

   (A) 120 rad/s (B) $\frac{\pi}{120}$ rad/s (C) 240 rad/s (D) $\frac{\pi}{240}$ rad/s (E) $\frac{120}{\pi}$ rad/s

2. Another Ferris Wheel at another amusement park has riders get on at position A, which is 3 meters above ground. The highest point of the ride is 15 m above the ground. The ride takes 40 seconds for one complete revolution.



I. An equation modeling the height of a rider over time for this Ferris Wheel is

(A) $h(t) = -6\cos 9t + 9$

(B) $h(t) = -6\cos 40\pi t + 9$

(C) $h(t) = -6\cos\dfrac{\pi t}{3} + 9$

(D) $h(t) = -6\cos\dfrac{\pi t}{20} + 9$

(E) $h(t) = -6\cos\dfrac{\pi t}{40} + 9$

II. The time it takes for a rider, who starts at position A, to travel to position B (a rotation of 135°) is

(A) 12 s (B) 13 s (C) 14 s (D) 15 s (E) 16 s

III. If the ride makes three complete rotations, the total amount of time a rider on the Ferris Wheel will spend above 13 m, rounded to the nearest second, is

(A) 11 s (B) 15 s (C) 25 s (D) 32 s (E) 45 s

IV. If the Ferris Wheel rotates counter-clockwise, instead of the original clockwise motion, the new equation can be represented by

(A) applying the transformation $y = -h(t)$

(B) applying the transformation $y = h(t-20)$

(C) applying the transformation $y = h(-t)$

(D) applying the transformation $y = -h(-t)$

(E) using a sine function instead of a cosine function, with no change to the parameters

Free Response

3. The pedals of a bicycle are mounted on a bracket whose center is 29 cm above the ground. Each pedal is 16.5 cm from the center of the bracket. Assume that the bicycle is pedaled at 12 revolutions per minute. With the starting position of pedal *A* in a horizontal position (heading down) at $t = 0$ seconds.

   (a) Sketch the graph of this sinusoidal function of the height of pedal *A* in cm (with respect to **seconds**) for the first three cycles. Label and number the axes appropriately.

   (b) Write an equation for this function expressing the height of pedal *A* y as a function of time t.

   (c) When is pedal *A* 40 cm above the ground for the first time?

   (d) How high is pedal *A* after 23 seconds? At this time, is the pedal going up or down?

4. The center of a bicycle wheel is 33 cm above the ground. A reflector mounted on the spokes is located 17.5 cm from the center of the wheel that is turning at a constant rate of one revolution per second. If y is the height of the reflector in cm, and t represents the time in seconds;

   (a) What is the amplitude of the reflector?

   (b) What is the period? How many rotations does the reflector make in 2π seconds?

   (c) If at $t = \frac{3}{4}$ seconds, the reflector is 33 cm above ground and moving up, what is the equation representing height of the reflector above ground in cm as a function of time in seconds?

   (d) Sketch <u>and label</u> this sinusoidal graph for two complete revolutions.

   (e) How high was the reflector when the bicycle first started moving at $t = 0$? At his time, was the reflector moving up or down?

   (f) At what time does the reflector first pass through a height of 25cm above the ground?

   (g) How would the equation change if the reflector was in a position 2 cm further from the center of the wheel?

5. The depth of water at the end of a pier varies with the tides throughout the day. Today the high tide occurs at 4:15 a.m. with a depth of 5.2 m. The low tide occurs at 10:27 a.m. with a depth of 2.0 m.

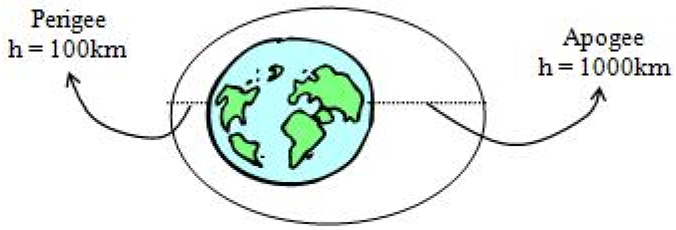
(a) Sketch a graph showing how the depth of the water, in meters, depends on the time, in hours, since midnight.

(b) Find a trigonometric equation that models the depth of the water, d, in meters, t hours after midnight.

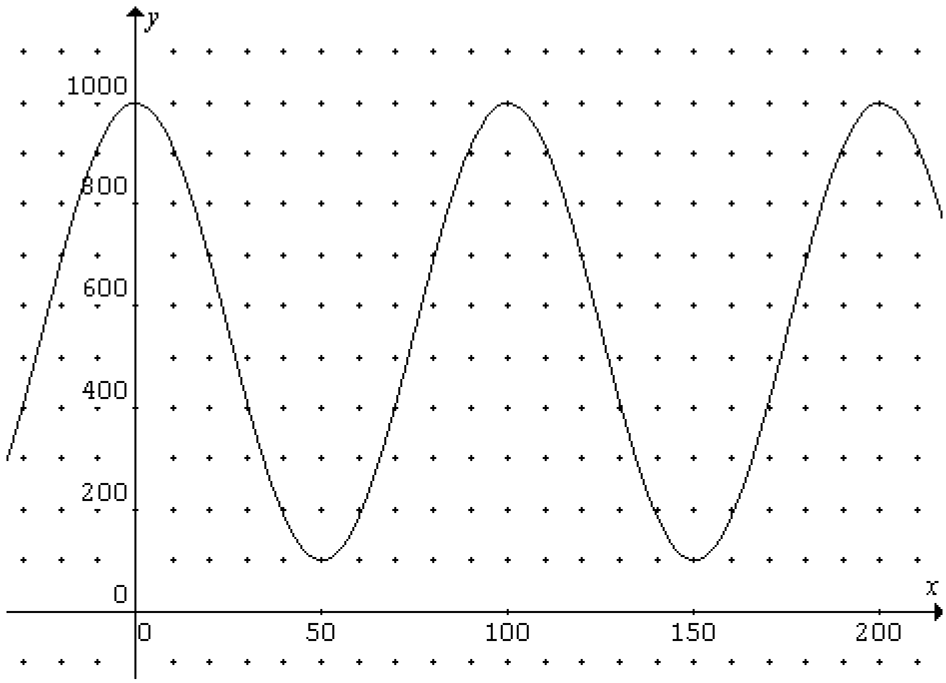
(c) Find the depth of the water at noon.

(d) A large boat needs at least 3 meters of water to moor at the end of the pier. To the nearest minute, during what times of day (clock time) after 12:00 noon can it safely moor?

6. A spacecraft is in an elliptical orbit around Earth as shown in the diagram.



At time $t = 0$ hours, the ship is at its **apogee** *(highest point)* $y = 1000$ km above the Earth's surface. Fifty minutes later, it's at its **perigee** *(lowest point)* $y = 100$ km. The sketch below shows this sinusoidal relationship.



(a) Write an equation for the height h of the ship, in km, above Earth in terms of t minutes.

(b) In order to transmit data to Earth, the ship must be within 700 km of the Earth's surface. For how many consecutive minutes will the spacecraft be able to transmit?

(c) The spacecraft is vulnerable to attack from ground missiles for a 20 minute interval while it is traveling through its lowest orbit. What is the maximum height these missiles can reach?

7. As Norman drives into his garage at night, a tiny stone becomes wedged between the treads in one of his tires. As he drives to work the next morning in his Toyota Corolla at a steady 35 mph, the distance of the stone from the pavement varies sinusoidally with the distance he travels, with the period being the circumference of his tire. Assume that his wheel has a radius of 12 inches and that at $t = 0$, the stone is at the bottom.

(a) Sketch a graph of the height of the stone, *h*, above the pavement, in inches, with respect to *x*, the distance the car travels down the road in inches. (Leave π visible on your *x*-axis).

(b) Determine the equation that most closely models the graph of $h(x)$ from part (a).

(c) How far will the car have traveled, in inches, when the stone is 9 inches from the pavement for the TENTH time?

(d) If Norman drives precisely 3 miles from his house to work, how high is the stone from the pavement when he gets to work? Was it on its way up or down? How can you tell?

(e) What kind of car does Norman drive?

8. On the very next day, Norman goes to work again, this time in his equally fuel-efficient Toyota Camry. The Camry also has a stone wedged in its tires, which have a 12 inch radius as well. As he drives to work in his Camry at a predictable, steady, smooth, consistent 35 mph, the distance of the stone from the pavement varies sinusoidally with the time he spends driving to work with the period being the time it takes for the tire to make one complete revolution. **When Norman begins this time, at $t = 0$ seconds, the stone is 3 inches above the pavement heading down.**

(a) Sketch a graph of the stone's distance from the pavement $h(t)$, in inches, as a function of time t, in seconds. Show at least one cycle and at least one critical value less than zero.

(b) Determine the equation that most closely models the graph of $h(t)$.

(c) How much time has passed when the stone is 16 inches from the pavement going TOWARD the pavement for the EIGHTH time?

(d) If Norman drives precisely 3 miles from his house to work, how high is the stone from the pavement when he gets to work? Was it on its way up or down?

(e) If Norman is driving to work with his cat in the car, in what kind of car is Norman's cat riding?