

Name _____ Date _____ Period _____

Xtra Practice: Proofs and Trig equations**Part I: Prove the following identities. Show all steps working down vertically. Each line must be an equivalent statement to the one above it.**

1. $1 + \frac{\cot A}{\csc A} - \sin^2 A = \cos A(\cos A + 1)$

2. $\tan B + \cot B = \csc B \sec B$

3. $\frac{\cot \phi}{\sec \phi} = \csc \phi - \sin \phi$

4. $\frac{\sec \beta}{1 + \cos \beta} = \csc^2 \beta (\sec \beta - 1)$

5. $(\tan \alpha - \sec \alpha)^2 = \frac{1 - \sin \alpha}{1 + \sin \alpha}$

6. $\sin^4 \psi - \cos^4 \psi = 1 - 2\cos^2 \psi$

7. $\sec^4 \delta - 2\sec^2 \delta \tan^2 \delta + \tan^4 \delta = 1$

8. $\sqrt[3]{\tan^2 x - \sec^2 x} = -1$

9. $\frac{1}{\csc y + \cot y} = \frac{1 - \cos y}{\sin y}$

10. $\frac{\sin \kappa - 1}{\cos \kappa} = \tan \kappa - \sec \kappa$

11. $\frac{\sec \omega}{\sin \omega} - \frac{\sin \omega}{\cos \omega} = \cot \omega$

Part II: Solve

12. Find the exact values for $\sqrt{3} \cot x - 1 = 0$ $x \in [0, 2\pi)$ (hint: turn it into a $\tan x$ problem)

Part III: Review

Determine the exact values of the other five trigonometric functions for θ . Given the following information. Draw the reference triangles.

$$13. \cos \theta = -\frac{7}{13}; \frac{\pi}{2} < \theta < \pi$$

$$14. \sin \theta = -\frac{4}{5}; \frac{3\pi}{2} < \theta < 2\pi$$

Part IV:

Determine whether the following equations are true or false. Show the final values you used in making your decision.

$$15. \tan^2(-\pi) + \sec \frac{3\pi}{4} = \sqrt{\left[\cos^2\left(\frac{5\pi}{6}\right) + \sin^2\left(\frac{5\pi}{6}\right) \right]}$$

$$16. \csc \frac{5\pi}{6} + \sec\left(-\frac{\pi}{3}\right) = \left[\cot^2\left(\frac{3\pi}{4}\right) + \tan^2\left(\frac{7\pi}{4}\right) \right]^2$$

Part V: Additional Practice:

Prove each identity.

17. $\cos^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta$

18. $(1 - \sin^2 \theta)(1 + \tan^2 \theta) = 1$

19. $\frac{1}{1 + \sin \phi} + \frac{1}{1 - \sin \phi} = 2 \sec^2 \phi$

20. $\cos^2 \theta - \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

21. $(\csc \beta - \sin \beta)^2 = \cot^2 \beta - \cos^2 \beta$

22. $\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$

23.
$$\frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}$$

24.
$$\frac{\cot x}{\csc x + 1} = \frac{\csc x - 1}{\cot x}$$

Part VI: Simplify each expression (as a single trig function without a fraction, if possible)

25. $\sin^2 \theta + \cos 2\theta$

26. $\sec^4 x - \tan^4 x - \tan^2 x$

27.
$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$$

28.
$$\frac{1 + \tan \phi}{\sin \phi} - \sec \phi$$

Part VII: Find a counterexample to show that each statement is not an identity.

29. $\cos 2\theta = 2\cos \theta$ 30. $\sin(\theta - \beta) = \sin \theta - \sin \beta$ 31. $\tan \frac{1}{2}\theta = \frac{1}{2}\tan \theta$ 32. $\sqrt{\tan^2 \theta + 1} = \sec \theta$