PreCal Matters: Xtra Practice: Proofs and Trig Eqs

I.

① $1+\frac{\cot A}{\csc A}-\sin^2 A = \cos A(\cos A+1)$

$1+\frac{\cos A}{\sin A}\cdot \sin A-\sin^2 A$

$1-\sin^2 A+\cos A$

$\cos^2 A+\cos A$

$\cos A(\cos A+1)$

- done

② $\tan B+\cot B = \csc B\sec B$

$\frac{\sin B}{\cos B}+\frac{\cos B}{\sin B}$

$\frac{\sin^2 B+\cos^2 B}{\sin B\cos B}$

$\frac{1}{\sin B\cos B}$

$\csc B\sec B$

- done

③ $\frac{\cot\phi}{\sec\phi} = \csc\phi-\sin\phi$

$\frac{1}{\sin\phi}-\frac{\sin\phi}{1}$

$\frac{1-\sin^2\phi}{\sin\phi}$

$\frac{\cos^2\phi}{\sin\phi}$

$\frac{\cos\phi}{\sin\phi}\cdot\frac{\cos\phi}{1}$

$\cot\phi\left(\frac{1}{\sec\phi}\right)$

$\frac{\cot\phi}{\sec\phi}$

- done

④ $\frac{\sec\beta}{1+\cos\beta} = \csc^2\beta(\sec\beta-1)$

$=\frac{1}{\sin^2\beta}\left(\frac{1}{\cos\beta}-1\right)$

$=\frac{1}{\sin^2\beta\cos\beta}-\frac{1}{\sin^2\beta}\left(\frac{\cos\beta}{\cos\beta}\right)$

$=\frac{1-\cos\beta}{(\sin^2\beta)\cdot\cos\beta}$

$=\frac{1-\cos\beta}{(1-\cos^2\beta)\cos\beta}$

$=\frac{\cancel{(1-\cos\beta)}}{\cancel{(1-\cos\beta)}(1+\cos\beta)\cos\beta}$

$=\frac{\sec\beta}{1+\cos\beta}$

- done

⑤ $(\tan\alpha-\sec\alpha)^2 = \frac{1-\sin\alpha}{1+\sin\alpha}$

$\left(\frac{\sin\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}\right)^2$

$\left(\frac{\sin\alpha-1}{\cos\alpha}\right)^2$

$\frac{\sin^2\alpha-2\sin\alpha+1}{\cos^2\alpha}$

$\frac{1-2\sin\alpha+\sin^2\alpha}{1-\sin^2\alpha}$

$\frac{(1-\sin\alpha)^2}{(1-\sin\alpha)(1+\sin\alpha)}$

$\frac{\cancel{(1-\sin\alpha)}(1-\sin\alpha)}{\cancel{(1-\sin\alpha)}(1+\sin\alpha)}$

$\frac{1-\sin\alpha}{1+\sin\alpha}$

- done

⑥ $\sin^4\psi-\cos^4\psi = 1-2\cos^2\psi$

$(\sin^2\psi-\cos^2\psi)(\sin^2\psi+\cos^2\psi)$

$((1-\cos^2\psi)-\cos^2\psi)\cdot 1$

$1-2\cos^2\psi$

- done

⑦ $\sec^4\delta-2\sec^2\delta\tan^2\delta+\tan^4\delta = 1$

$(\sec^2\delta-\tan^2\delta)^2$

$((1+\tan^2\delta)-\tan^2\delta)^2$

1^2

1

- done

(8) $\sqrt[3]{\tan^2 x - \sec^2 x} \neq -1$

$\sqrt[3]{(\sec^2 x - 1) - \sec^2 x}$

$\sqrt[3]{-1}$

-1 — done

(9) $\dfrac{1}{\csc y + \cot y} \neq \dfrac{1-\cos y}{\sin y}$

$\dfrac{1}{\frac{1}{\sin y} + \frac{\cos y}{\sin y}}$ | $\dfrac{1-\cos y}{\sin y}\left(\dfrac{1+\cos y}{1+\cos y}\right)$

$\dfrac{1}{\frac{1+\cos y}{\sin y}}$ | $\dfrac{1-\cos^2 y}{\sin y(1+\cos y)}$

$\dfrac{\sin y}{1+\cos y}$ | $\dfrac{\cancel{\sin y}\cdot \sin y}{\cancel{\sin y}(1+\cos y)}$

~ done | $\dfrac{\sin y}{1+\cos y}$

(10) $\dfrac{\sin K - 1}{\cos K} \neq \tan K - \sec K$

$\dfrac{\sin K}{\cos K} - \dfrac{1}{\cos K}$

— done | $\dfrac{\sin K - 1}{\cos K}$

(11) $\dfrac{\sec w}{\sin w} - \dfrac{\sin w}{\cos w} \neq \cot w$

$\dfrac{\sec w \cos w - \sin^2 w}{\sin w \cos w}$

$\dfrac{1-\sin^2 w}{\sin w \cos w}$

$\dfrac{\cos w \cdot \cancel{\cos w}}{\sin w \cdot \cancel{\cos w}}$ — done

$\cot w$

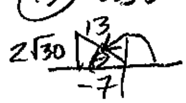
II. (12) $\sqrt{3}\cot x - 1 = 0,\ x \in [0, 2\pi)$

$\cot x = \frac{1}{\sqrt{3}}$

$\tan x = \dfrac{\sqrt{3}/2}{1/2}$

$\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$

III. (13) $\cos\theta = -\frac{7}{13},\ \frac{\pi}{2} < \theta < \pi$

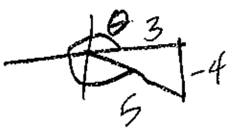


$\sin\theta = \frac{2\sqrt{30}}{13}$ | $\csc\theta = \frac{13}{2\sqrt{30}} = \frac{13\sqrt{30}}{60}$

$\cos\theta = -\frac{7}{13}$ | $\sec\theta = -\frac{13}{7}$

$\tan\theta = -\frac{2\sqrt{30}}{7}$ | $\cot\theta = -\frac{7}{2\sqrt{30}} = -\frac{7\sqrt{30}}{60}$

(14) $\sin\theta = -\frac{4}{5},\ \frac{3\pi}{2} < \theta < 2\pi$



$\sin\theta = -\frac{4}{5}$ | $\csc\theta = -\frac{5}{4}$

$\cos\theta = \frac{3}{5}$ | $\sec\theta = \frac{5}{3}$

$\tan\theta = -\frac{4}{3}$ | $\cot\theta = -\frac{3}{4}$

IV (15) $\tan^2(-\pi) + \sec\frac{3\pi}{4} = \sqrt{\cos^2\left(\frac{5\pi}{6}\right) + \sin^2\left(\frac{5\pi}{6}\right)}$

$0^2 + -\sqrt{2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$\boxed{-\sqrt{2} \neq 1}$

(16) $\csc\frac{5\pi}{6} + \sec\left(-\frac{\pi}{3}\right) = \left[\cot^2\left(\frac{3\pi}{4}\right) + \tan^2\left(\frac{7\pi}{4}\right)\right]^2$

$2 + \frac{1}{2} = [1 + 1]^2$

$\boxed{\frac{5}{2} \neq 4}$

V.

(17) $\cos^2\theta - \sin^2\theta = 1-2\sin^2\theta$

$(1-\sin^2\theta) - \sin^2\theta$

$1-2\sin^2\theta$ — done

(18) $(1-\sin^2\theta)(1+\tan^2\theta) = 1$

$\cos^2\theta \cdot \sec^2\theta$

$(\cos\theta \cdot \sec\theta)^2$

1^2

1 — done

(19) $\dfrac{1}{1+\sin\phi} + \dfrac{1}{1-\sin\phi} = 2\sec^2\phi$

$\dfrac{1-\sin\phi+1+\sin\phi}{(1+\sin\phi)(1-\sin\phi)}$

$\dfrac{2}{1-\sin^2\phi}$

$\dfrac{2}{\cos^2\phi}$ — done

$2\sec^2\phi$

(20) $\cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\dfrac{1-\tan^2\theta}{\sec^2\theta}$

$(1-\tan^2\theta)\cos^2\theta$

$\cos^2\theta - \cancel{\cos^2\theta}\left(\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}\right)$

$\cos^2\theta - \sin^2\theta$

— done

(21) $(\csc\beta - \sin\beta)^2 = \cot^2\beta - \cos^2\beta$

$\left(\dfrac{1}{\sin\beta} - \dfrac{\sin\beta}{1}\right)^2$ | $\dfrac{\cos^2\beta}{\sin^2\beta} - \dfrac{\cos^2\beta}{1}$

$\left(\dfrac{1-\sin^2\beta}{\sin\beta}\right)^2$ | $\dfrac{\cos^2\beta - \cos^2\beta\sin^2\beta}{\sin^2\beta}$

$\left(\dfrac{\cos^2\beta}{\sin\beta}\right)^2$ | $\dfrac{\cos^2\beta(1-\sin^2\beta)}{\sin^2\beta}$

$\dfrac{\cos^4\beta}{\sin^2\beta}$ | $\dfrac{\cos^4\beta}{\sin^2\beta}$

(22) $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{1-\sin\theta}{\cos\theta} = 2\sec\theta$

$\dfrac{\cos^2\theta + (1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$

$\dfrac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)}$

$\dfrac{1+1-2\sin\theta}{\cos\theta(1-\sin\theta)}$

$\dfrac{2\cancel{(1-\sin\theta)}}{\cos\theta\cancel{(1-\sin\theta)}}$ — done

$2\sec\theta$

(23) $\dfrac{\tan^2 x}{\sec x+1} = \dfrac{1-\cos x}{\cos x}$

$\dfrac{\sec^2 x - 1}{\sec x + 1}$

$\dfrac{(\sec x-1)\cancel{(\sec x+1)}}{\cancel{(\sec x+1)}}$

$\dfrac{1}{\cos x} - 1$

$\dfrac{1-\cos x}{\cos x}$ — done

(24) $\dfrac{\cot x}{\csc x+1} = \dfrac{\csc x - 1}{\cot x}\left(\dfrac{\csc x+1}{\csc x+1}\right)$

$\dfrac{\csc^2 x - 1}{\cot x(\csc x+1)}$

$\dfrac{\cancel{\cot x}\cdot\cot x}{\cancel{\cot x}(\csc x+1)}$

— done $\dfrac{\cot x}{\csc x+1}$

VI. (25) $\sin^2\theta + \cos 2\theta$

$\frac{1}{2}(1-\cos 2\theta)+\cos 2\theta$

$\frac{1}{2}-\frac{1}{2}\cos 2\theta+\cos 2\theta$

$\boxed{\frac{1}{2}+\frac{1}{2}\cos 2\theta}$

(26) $\sec^4 x - \tan^4 x - \tan^2 x$

$(\sec^2 x-\tan^2 x)(\sec^2 x+\tan^2 x)-\tan^2 x$

$1\cdot(\sec^2 x+\tan^2 x)-\tan^2 x$

$\sec^2 x + 0$

$\boxed{\sec^2 x}$

(27) $\dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}$

$\dfrac{1}{\cos^2\theta}$

$\boxed{\sec^2\theta}$

(28) $\dfrac{1+\tan\phi}{\sin\phi}-\sec\phi$

$\left(\dfrac{\cos\phi}{\cos\phi}\right)\dfrac{1+\frac{\sin\phi}{\cos\phi}}{\sin\phi}-\dfrac{1}{\cos\phi}$

$\dfrac{\cos\phi+\sin\phi}{\sin\phi\cos\phi}-\dfrac{1}{\cos\phi}\left(\dfrac{\sin\phi}{\sin\phi}\right)$

$\dfrac{\cos\phi+\sin\phi-\sin\phi}{\sin\phi\cos\phi}$

$\dfrac{\cancel{\cos\phi}}{\sin\phi\cancel{\cos\phi}}$

$\boxed{\csc\phi}$

VII.

(29) $\cos 2\theta \neq 2\cos\theta,\ \theta=\frac{\pi}{2}$

$\cos\left(2\left(\frac{\pi}{2}\right)\right) \quad | \quad 2\cos\frac{\pi}{2}$

$\cos\pi \quad | \quad 2(0)$

$-1 \neq 0$

(30) $\sin(\theta-\beta) \neq \sin\theta-\sin\beta,\ \theta=\frac{\pi}{2}, \beta=\frac{\pi}{4}$

$\sin\left(\frac{\pi}{2}-\frac{\pi}{4}\right) \quad | \quad \sin\frac{\pi}{2}-\sin\frac{\pi}{4}$

$\sin\frac{\pi}{4} \quad | \quad 1-\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} \neq \frac{2-\sqrt{2}}{2}$

(31) $\tan\frac{1}{2}\theta \neq \frac{1}{2}\tan\theta,\ \theta=\frac{\pi}{2}$

$\tan\left(\frac{1}{2}\left(\frac{\pi}{2}\right)\right) \quad | \quad \frac{1}{2}\tan\frac{\pi}{2}$

$\tan\frac{\pi}{4} \quad | \quad$ DNE

$1 \neq$

(32) $\sqrt{\tan^2\theta+1} \neq \sec\theta$ (Choose an angle in QII or QIII)

$\theta=\frac{2\pi}{3}$ where $\sec\theta<0$

$\sqrt{\left(\tan\frac{2\pi}{3}\right)^2+1} \quad | \quad \sec\frac{2\pi}{3}$

$\sqrt{(-\sqrt{3})^2+1} \quad | \quad -2$

$\sqrt{3+1}$

$2 \neq$