Name____ Date________________________ Period______

Worksheet 8.2—The Integral

Show all work on a separate sheet of paper. **No Calculator** unless otherwise specified.

For the following graphs of functions $y = f(x)$, evaluate geometrically. Be sure to sketch the graph and identify the region. Verify 1, 2, and 3 by using the FTOC.

1. $\int_{-2}^{3} 4dx$ 2. $\int_{0}^{2} (4-2x)dx$ 3. $\int_{0}^{1} (2x+3)dx$ 4. $\int_{0}^{3} \sqrt{9-x^2}\,dx$ (a semicircle)

Find the area of the indicated region by setting up a definite integral, then using the FTOC. Be sure to sketch the graph and identify the region. Bounded by . . .

5. $y = e^x$ and $y = 0$ from $x = \ln 2$ to $x = \ln 7$ 6. $f(x) = x^{-1} + 3$ and the x-axis on the interval $x \in [1,4]$

7. $g(x) = 2x^3 + 3x^2 + 1$, $x = 0$, and $x = 1$ 8. $y = \sin t$, $y = 0$, $\frac{\pi}{6} \le t \le \frac{2\pi}{3}$

Evaluate the following indefinite integrals by finding the general antiderivative. Don't forget your +*C*. You may have to simplify/rewrite prior to integrating.

9. $\int \left(4\sqrt{x} - \frac{5}{4}\sqrt[3]{x} + \frac{2}{\sqrt[4]{x^5}} \right) dx$ 10. $\int 2x^2 (3x-1)^2 dx$ 11. $\int \left(\frac{x\sin x - 6xe^x + 2\sqrt[4]{x} + 1}{x} \right) dx$

Challenge yourself by trying to evaluate the following indefinite integrals. Don't forget your +*C*. Check your answers by differentiating.

12. $\int \cos(2x)\,dx$ 13. $\int (5x-7)^{10} dx$ 14. If $f'(x) = xe^{x^2}$, find the general antiderivative $f(x)$.

A **particular solution** to an indefinite integral (or **differential equation**) has a unique value of *C*. To find this value of *C*, we need a point through which the graph of our particular solution passes. This given point is called an **initial condition**. Once the particular solution is found, other points on the graph of the solution curve can be found.

15. (a) Sketch the graph of the differential function $y' = 2 + \frac{1}{x^2}$ on the interval $x \in [0,3]$

(b) Find the particular solution to the differential equation $y' = 2 + \frac{1}{x^2}$ by setting up and evaluating an indefinite integral.

(c) If $y(1) = 6$, find the particular solution to the differential equation from part (b).

(d) Find the equation of the tangent line for the function $y = f(x)$ from part (c) at $x = 1$.

(e) Evaluate $y(2)$ using your particular solution from part (c).

(f) Approximate $y(2)$ by using the tangent line equation from part (d). Can you explain why this number is different but similar to your answer from part (e)?

(g) Finally, evaluate the integral expression: $6 + \int_{1}^{2} \left(2 + \frac{1}{x^2} \right) dx$. What do you notice? How does it compare to your answer from part (e)? Can you explain this geometrically? Try. Write down your thoughts and draw a picture, if necessary, to aid your explanation.