

## EXERCISES

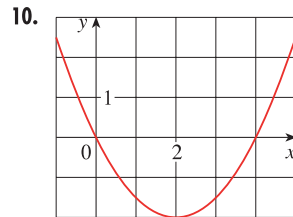
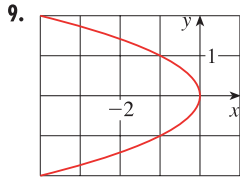
**A** [Click here for answers.](#)

**S** [Click here for solutions.](#)

**1–8** ■ Find the vertex, focus, and directrix of the parabola and sketch its graph.

1.  $x = 2y^2$
2.  $4y + x^2 = 0$
3.  $4x^2 = -y$
4.  $y^2 = 12x$
5.  $(x + 2)^2 = 8(y - 3)$
6.  $x - 1 = (y + 5)^2$
7.  $y^2 + 2y + 12x + 25 = 0$
8.  $y + 12x - 2x^2 = 16$

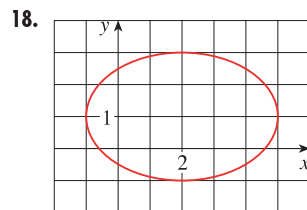
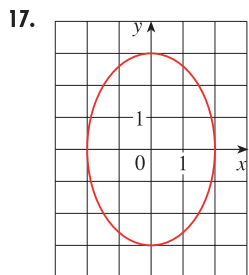
**9–10** ■ Find an equation of the parabola. Then find the focus and directrix.



**11–16** ■ Find the vertices and foci of the ellipse and sketch its graph.

11.  $\frac{x^2}{9} + \frac{y^2}{5} = 1$
12.  $\frac{x^2}{64} + \frac{y^2}{100} = 1$
13.  $4x^2 + y^2 = 16$
14.  $4x^2 + 25y^2 = 25$
15.  $9x^2 - 18x + 4y^2 = 27$
16.  $x^2 + 2y^2 - 6x + 4y + 7 = 0$

**17–18** ■ Find an equation of the ellipse. Then find its foci.



**19–20** ■ Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph.

19.  $\frac{x^2}{144} - \frac{y^2}{25} = 1$
20.  $\frac{y^2}{16} - \frac{x^2}{36} = 1$
21.  $y^2 - x^2 = 4$
22.  $9x^2 - 4y^2 = 36$

23.  $2y^2 - 3x^2 - 4y + 12x + 8 = 0$

24.  $16x^2 - 9y^2 + 64x - 90y = 305$

**25–30** ■ Identify the type of conic section whose equation is given and find the vertices and foci.

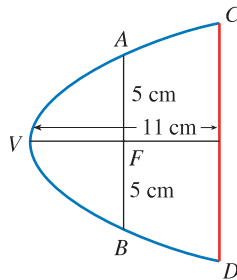
25.  $x^2 = y + 1$
26.  $x^2 = y^2 + 1$
27.  $x^2 = 4y - 2y^2$
28.  $y^2 - 8y = 6x - 16$
29.  $y^2 + 2y = 4x^2 + 3$
30.  $4x^2 + 4x + y^2 = 0$

**31–48** ■ Find an equation for the conic that satisfies the given conditions.

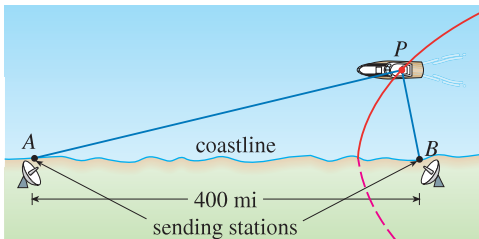
31. Parabola, vertex (0, 0), focus (0, -2)
32. Parabola, vertex (1, 0), directrix  $x = -5$
33. Parabola, focus (-4, 0), directrix  $x = 2$
34. Parabola, focus (3, 6), vertex (3, 2)
35. Parabola, vertex (0, 0), axis the  $x$ -axis, passing through (1, -4)
36. Parabola, vertical axis, passing through (-2, 3), (0, 3), and (1, 9)
37. Ellipse, foci ( $\pm 2$ , 0), vertices ( $\pm 5$ , 0)
38. Ellipse, foci (0,  $\pm 5$ ), vertices (0,  $\pm 13$ )
39. Ellipse, foci (0, 2), (0, 6) vertices (0, 0), (0, 8)
40. Ellipse, foci (0, -1), (8, -1), vertex (9, -1)
41. Ellipse, center (2, 2), focus (0, 2), vertex (5, 2)
42. Ellipse, foci ( $\pm 2$ , 0), passing through (2, 1)
43. Hyperbola, foci (0,  $\pm 3$ ), vertices (0,  $\pm 1$ )
44. Hyperbola, foci ( $\pm 6$ , 0), vertices ( $\pm 4$ , 0)
45. Hyperbola, foci (1, 3) and (7, 3), vertices (2, 3) and (6, 3)
46. Hyperbola, foci (2, -2) and (2, 8), vertices (2, 0) and (2, 6)
47. Hyperbola, vertices ( $\pm 3$ , 0), asymptotes  $y = \pm 2x$
48. Hyperbola, foci (2, 2) and (6, 2), asymptotes  $y = x - 2$  and  $y = 6 - x$

49. The point in a lunar orbit nearest the surface of the moon is called *perilune* and the point farthest from the surface is called *apolune*. The *Apollo 11* spacecraft was placed in an elliptical lunar orbit with perilune altitude 110 km and apolune altitude 314 km (above the moon). Find an equation of this ellipse if the radius of the moon is 1728 km and the center of the moon is at one focus.

50. A cross-section of a parabolic reflector is shown in the figure. The bulb is located at the focus and the opening at the focus is 10 cm.
- Find an equation of the parabola.
  - Find the diameter of the opening  $|CD|$ , 11 cm from the vertex.



51. In the LORAN (LONG RANGE Navigation) radio navigation system, two radio stations located at  $A$  and  $B$  transmit simultaneous signals to a ship or an aircraft located at  $P$ . The onboard computer converts the time difference in receiving these signals into a distance difference  $|PA| - |PB|$ , and this, according to the definition of a hyperbola, locates the ship or aircraft on one branch of a hyperbola (see the figure). Suppose that station  $B$  is located 400 mi due east of station  $A$  on a coastline. A ship received the signal from  $B$  1200 microseconds ( $\mu\text{s}$ ) before it received the signal from  $A$ .
- Assuming that radio signals travel at a speed of  $980 \text{ ft}/\mu\text{s}$ , find an equation of the hyperbola on which the ship lies.
  - If the ship is due north of  $B$ , how far off the coastline is the ship?



- Use the definition of a hyperbola to derive Equation 6 for a hyperbola with foci  $(\pm c, 0)$  and vertices  $(\pm a, 0)$ .
- Show that the function defined by the upper branch of the hyperbola  $y^2/a^2 - x^2/b^2 = 1$  is concave upward.
- Find an equation for the ellipse with foci  $(1, 1)$  and  $(-1, -1)$  and major axis of length 4.
- Determine the type of curve represented by the equation

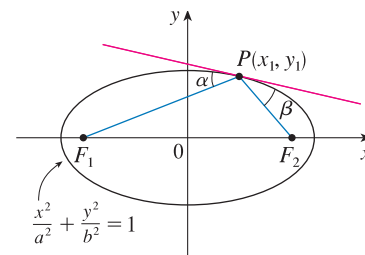
$$\frac{x^2}{k} + \frac{y^2}{k-16} = 1$$

- in each of the following cases: (a)  $k > 16$ , (b)  $0 < k < 16$ , and (c)  $k < 0$ .
- Show that all the curves in parts (a) and (b) have the same foci, no matter what the value of  $k$  is.

- Show that the equation of the tangent line to the parabola  $y^2 = 4px$  at the point  $(x_0, y_0)$  can be written as  $y_0y = 2p(x + x_0)$ .
  - What is the  $x$ -intercept of this tangent line? Use this fact to draw the tangent line.
57. Use Simpson's Rule with  $n = 10$  to estimate the length of the ellipse  $x^2 + 4y^2 = 4$ .
58. The planet Pluto travels in an elliptical orbit around the Sun (at one focus). The length of the major axis is  $1.18 \times 10^{10}$  km and the length of the minor axis is  $1.14 \times 10^{10}$  km. Use Simpson's Rule with  $n = 10$  to estimate the distance traveled by the planet during one complete orbit around the Sun.
59. Let  $P(x_1, y_1)$  be a point on the ellipse  $x^2/a^2 + y^2/b^2 = 1$  with foci  $F_1$  and  $F_2$  and let  $\alpha$  and  $\beta$  be the angles between the lines  $PF_1, PF_2$  and the ellipse as in the figure. Prove that  $\alpha = \beta$ . This explains how whispering galleries and lithotripsy work. Sound coming from one focus is reflected and passes through the other focus. [Hint: Use the formula

$$\tan \alpha = \frac{m_2 - m_1}{1 + m_2 m_1}$$

to show that  $\tan \alpha = \tan \beta$ . See [Challenge Problem 2.13](#).]



60. Let  $P(x_1, y_1)$  be a point on the hyperbola  $x^2/a^2 - y^2/b^2 = 1$  with foci  $F_1$  and  $F_2$  and let  $\alpha$  and  $\beta$  be the angles between the lines  $PF_1, PF_2$  and the hyperbola as shown in the figure. Prove that  $\alpha = \beta$ . (This is the reflection property of the hyperbola. It shows that light aimed at a focus  $F_2$  of a hyperbolic mirror is reflected toward the other focus  $F_1$ .)

