Name________________________________ Date____________________ Period______

Worksheet 4.4—Properties of Logs

Show all work on a separate sheet of paper. All answers must be given as either simplified, exact answers. No calculator is permitted unless otherwise stated.

Multiple Choice

1. $\log 12 =$ $\log(3\cdot 4)$
   (A) $3\log 4$ (B) $\log 3 + \log 4$ (C) $4\log 3$ (D) $\log 3 \cdot \log 4$ (E) $2\log 6$

2. $\log_9 64 =$ Change of base
   (A) $5\log_3 2$ (B) $(\log_3 8)^2$ (C) $\dfrac{\ln 64}{\ln 9}$ (D) $2\log_9 32$ (E) $\dfrac{\log 64}{9}$

3. $2^{-1}\cdot(-3\ln 2 - 1) =$ $\frac{1}{2}(-\ln 8 - 1) = \frac{1}{2}(-(\ln 8 + 1)) = \frac{1}{2}(-(\ln 8 + \ln e)) = -\frac{1}{2}\ln(8e)$
   (A) $-\dfrac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\dfrac{3}{2}\ln 2$ (D) $-\dfrac{1}{2}$ (E) $\dfrac{1}{8}$

4. $\log_{1/2} x^2 =$ $2\log_{\frac{1}{2}} x$ $\;2\left(\dfrac{\log_2 x}{\log_2 \frac{1}{2}}\right) = 2\left(\dfrac{\log_2 x}{\log_2 2^{-1}}\right) = 2\left(\dfrac{\log_2 x}{-1}\right) = -2(\log_2 x)$
   (A) $-2\log_2 x$ (B) $2\log_2 x$ (C) $-0.5\log_2 x$ (D) $0.5\log_2 x$ (E) $-2\log_2 |x|$

5. $\ln x^5 =$ $5\ln x$
   (A) $\dfrac{5\log_7 x}{\log_7 e}$ (B) $\dfrac{2\log x^3}{\log e}$ (C) $\dfrac{x\log_{1/2} 5}{\log_{1/2} e}$ (D) $3\ln x^2$ (E) $\ln x^2 \cdot \ln x^3$

Short Answer

6. Evaluate each of the following expressions using the properties of logs (and no calculator).
   (a) $\log_3 \sqrt[3]{81}$ (b) $\log 4 + \log 25$ (c) $\log_2 6 - \log_2 15 + \log_2 20$ (d) $\ln\left(\ln e^{e^{200}}\right)$

   (a) $= \log_3 3^{4/3}$ $= \frac{4}{3}$

   (b) $= \log 100$ $= 2$

   (c) $= \log_2 \frac{6(20)}{15}$ $= \log_2 8$ $= 3$

   (d) $= \ln(e^{200})$ $= 200$

7. Use the properties of logs to expand the following expressions.

(a) $\log_5 \sqrt[4]{x^3(x^2+1)}$ (b) $\log_6 \sqrt{\frac{5x^2y^3}{x^2+y^3}}$ (c) $\log\sqrt{x\sqrt{y\sqrt{z}}}$ (d) $\ln\left(\frac{7x^4\sqrt{x^4-7}}{e^2(x-5)^2\sqrt[3]{2-6x^2}}\right)$

a) $\log_5\left(x^3(x^2+1)\right)^{\frac{1}{4}}$

$\frac{3}{4}\log_5 x+\frac{1}{4}\log_5(x^2+1)$

b) $\frac{1}{2}\log_6 5+\log_6 x+\frac{3}{2}\log_6 y-\frac{1}{2}\log_6(x^2+y^3)$

c) $\frac{1}{2}\log x+\frac{1}{4}\log y+\frac{1}{8}\log z$

D) $\ln 7+4\ln x+\frac{1}{2}\ln(x^4-7)-2-2\ln(x-5)-\frac{1}{3}\ln(2-6x^2)$

8. Use the properties of logs to condense the following expressions.

(a) $4\ln x-\frac{1}{3}\ln(x^2+1)+2\ln(x-1)$ (b) $\frac{1}{3}\ln(2x+1)+\frac{1}{2}\left[\log(x-4)-\log(x^4-x^2-1)\right]$

(c) $\log(x^2-1)-\ln(x-1)$ (use the change of base formula on this one first)

a) $\ln\left(\frac{x^4(x-1)^2}{\sqrt[3]{x^2+1}}\right)$ b) $\ln\sqrt[3]{2x+1}+\log\frac{\sqrt{x-4}}{\sqrt{x^4-x^2-1}}$

c) $\frac{\ln(x^2-1)}{\ln 10}-\ln(x-1)=\frac{1}{\ln 10}\ln(x^2-1)-\ln(x-1)$

$\ln\left(\frac{(x^2-1)^{\frac{1}{\ln 10}}}{(x-1)}\right)$

9. If $\log_7 x = A\log_{2/3} x$, use the change of base formula to find the value of A,

$\log_7 x=\frac{\log_{2/3}x}{\log_{2/3}7}=\left(\frac{1}{\log_{2/3}7}\right)\log_{\frac{2}{3}}x$

so $A=\frac{1}{\log_{2/3}7}=\frac{1}{\frac{\ln 7}{\ln 2/3}}=\frac{\ln(2/3)}{\ln 7}$

≈ -0.208

$\log_7 x = -0.208\log_{\frac{2}{3}} x$

10. Simplify the following to a single log expression of the form $\log_b a$: $(\log_7 3)(\log_2 5)(\log_5 7)$

$$\frac{\log 3}{\cancel{\log 7}} \cdot \frac{\cancel{\log 5}}{\log 2} \cdot \frac{\cancel{\log 7}}{\cancel{\log 5}} = \frac{\log 3}{\log 2} = \boxed{\log_2 3}$$

11. Use the properties of logs to show that $-\ln\left(x-\sqrt{x^2-1}\right)=\ln\left(x+\sqrt{x^2-1}\right)$. You may want to eventually multiply be a clever form of one.

$$\ln\left(\frac{x-\sqrt{x^2-1}}{1}\right)^{-1}$$

$$\ln\left(\frac{1}{x-\sqrt{x^2-1}}\left(\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right)\right) = \ln\left(x+\sqrt{x^2-1}\right)$$

$\log_b x = \log_b y \Leftrightarrow x = y$

12. Let $A=\ln 3$ and $B=\ln 5$, write each of the following in terms of A and B.
(a) $\ln 15$ (b) $\ln 27$ (c) $\ln 75$ (d) $\ln 45$ (e) $\log_5\sqrt{27}$

a) $\ln(3\cdot 5)$
$\ln 3+\ln 5$
$A+B$

b) $\ln 3^3$
$3\ln 3$
$3A$

c) $\ln 5^2\cdot 3$
$2\ln 5+\ln 3$
$2B+A$

D) $\ln 3^2\cdot 5$
$2\ln 3+\ln 5$
$2A+B$

E) $\log_5\sqrt{27}$
$\frac{\ln\sqrt{27}}{\ln 5}$
$\frac{\ln 3^{\frac{3}{2}}}{\ln 5} = \frac{3/2\,A}{B} = \frac{3A}{2B}$

13. (Calculator Permitted) Solve the following equations graphically on your calculator. Be sure to report three decimals in your answers.
(a) $\ln x > \sqrt[3]{x}$ (b) $1.2^x \le \log_{1.2} x$

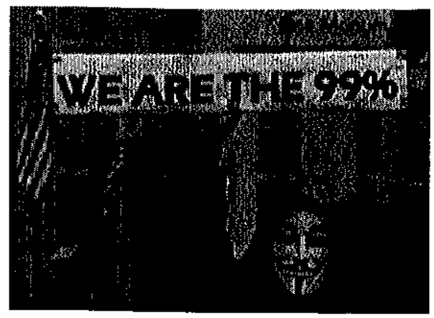
a) $\{x \mid 6.405 < x < 93.354\}$ b) $\{x \mid 1.258 x \le 14.767\}$

14. (Calculator Permitted) The "Occupy Wall Street" movement in 2011 is a protest against the unequal distribution of wealth in the United States. Vilfredo Pareto (1848-1923) observed that most of the wealth of any country is owned by a few members of the population. Pareto's Principle is given by

$$\log P = \log c - k \log W$$

Where W is the wealth level (how much money a person has) and P is the number of people in the population having that much money.

(a) Solve the equation for P.
(b) Assume $k = 2.1$, $c = 8000$, and W is measured in millions of dollars. Use part (a) to find the number of people who have \$2 million or more. How many people have \$10 million or more? If the population of the US is considered to be 312 million people, what percentage of the US population has \$10 million or more?



a) $\log P = \log \frac{c}{w^k}$

$P = 10^{\log \frac{c}{w^k}}$

$\boxed{P = \frac{c}{w^k}}$

b) $P = \frac{8000}{2^{2.1}}$

P: 1866 people

c)

$P = \frac{8000}{10^{2.1}} = 63.54$ people $= 0.0000002037 =$

0.00000002037%