Precal Matters, WS 6.4 KEY

① $f(x)=\sin x,\ g(x)=\cos x,\ f(2x)=\sin 2x = 2\sin x\cos x = 2f(x)\cdot g(x)$

$\boxed{D}$

② $\sin 22.5° = \sin\left(\frac{1}{2}(45°)\right) = +\sqrt{\frac{1}{2}(1-\cos 45°)} = \sqrt{\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

$\boxed{E}$

③ $\sin 2x = \cos x,\ 2\sin x\cos x - \cos x = 0,\ \cos x(2\sin x - 1) = 0,\ \cos x = 0 \text{ or } \sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{E}$

④ $f(x) = 3\sin^2 x - 3\cos^2 x,\ f(x) = -3(\cos^2 x - \sin^2 x),\ f(x) = -3\cos 2x$

$B=2$ so $P = \frac{2\pi}{2} = \pi$

$\boxed{C}$

⑤ $2\cos x + \sin 2x = 0,\ 2\cos x + 2\sin x\cos x = 0;\ 2\cos x(1+\sin x) = 0$

$\cos x = 0$ or $\sin x = -1$, $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{A}$

⑥ $\sin 2x - \cos 3x,\ 2\sin x\cos x - \cos(2x+x),\ 2\sin x\cos x - \cos 2x\cos x + \sin 2x\sin x,$

$2\sin x\cos x - \cos x(\cos^2 x - \sin^2 x) + 2\sin^2 x\cos x,\ 2\sin x\cos x - \cos^3 x + \underline{\cos x\sin^2 x} + 2\sin^2 x\cos x,$

$2\sin x\cos x - \underline{\cos^3 x} + 3\cos x\sin^2 x,\ 2\sin x\cos x - \cos x(1-\sin^2 x) + 3\cos x\sin^2 x,$

$2\sin x\cos x - \cos x + \cos x\sin^2 x + 3\cos x\sin^2 x,\ \underline{\underline{2\sin x\cos x - \cos x + 4\cos x\sin^2 x}}$

$\boxed{C}$

⑦ $\cos^2\left(\frac{x}{2}\right) = \cos^2 x,\ \frac{1}{2}(1+\cos x) = \cos^2 x,\ 1+\cos x = 2\cos^2 x,\ 2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0,\ \cos x = -\frac{1}{2} \text{ or } \cos x = 1,\ \boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0}$

$\boxed{E}$

⑧

(a) $\sin(3x) = 1$

$3x = \sin^{-1} 1$

$3x = \frac{\pi}{2} + 2\pi n$

$x = \frac{\pi}{6} + \frac{2\pi}{3}n$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

(b) $\sin 2x + \tan x = 0$

$\frac{2\sin x\cos x}{1} - \frac{\sin x}{\cos x} = 0$

$\frac{2\sin x\cos^2 x - \sin x}{\cos x} = 0 \leftarrow (\cos x \neq 0)$

when $\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0$ or $\cos^2 x = \frac{1}{2}$

$x = 0, \pi$ $\quad \cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\boxed{x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

since none of these make cosine = 0, they are in domain

(c) $\ln 1 - \cos x = \cos 2x$

$0 - \cos x = 2\cos^2 x - 1$

$0 = 2\cos^2 x + \cos x - 1$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2},\ \cos x = -1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi}$

(8) (d) $\cos 2x + \cos 4x = 0$

$\cos 2x + \cos(2(2x)) = 0$

$\cos 2x + 2\cos^2 2x - 1 = 0$

$2\cos^2(2x) + \cos(2x) - 1 = 0$

$(2\cos 2x - 1)(\cos(2x) + 1) = 0$

$\cos(2x) = \frac{1}{2}, \quad \cos(2x) = -1$

$2x = \cos^{-1}\left(\frac{1}{2}\right)$

$\begin{cases} 2x = \frac{\pi}{3} + 2\pi n \\ 2x = \frac{5\pi}{3} + 2\pi n \end{cases}$

$\begin{cases} x = \frac{\pi}{6} + \pi n \\ x = \frac{5\pi}{6} + \pi n \end{cases}$

$\boxed{x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}}$

$2x = \pi + 2\pi n$

$x = \frac{\pi}{2} + \pi n$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

(e) $\cos^2 x = \sin^2\left(\frac{x}{2}\right)$

$\cos^2 x = \frac{1}{2}(1 - \cos x)$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}, \quad \cos x = -1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi}$

(f) $\cos^2\left(\frac{1}{2}x\right) = 1 - \sin^2 x$

$\frac{1}{2}(1 + \cos x) = \cos^2 x$

$1 + \cos x = 2\cos^2 x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}, \quad \cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0}$

(9) (a) $\cos\frac{3\pi}{8} = \cos\left(\frac{1}{2}\left(\frac{3\pi}{4}\right)\right)$

$\left(\frac{3\pi}{8} \to QI \to POS\right)$

$= +\sqrt{\frac{1}{2}\left(1 + \cos\frac{3\pi}{4}\right)}$

$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}\left(\frac{2}{2}\right)}$

$= \sqrt{\frac{2 - \sqrt{2}}{4}}$

$= \boxed{\frac{\sqrt{2 - \sqrt{2}}}{2}}$

(b) $\tan 22.5° = \tan\left(\frac{1}{2}(45°)\right)$

$(22.5° \to QI \to POS)$

$\sqrt{\frac{1 - \cos 45°}{1 + \cos 45°}}$

$\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}}\left(\frac{2}{2}\right)}$

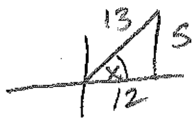
$\sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}\left(\frac{2 - \sqrt{2}}{2 - \sqrt{2}}\right)}$

$\sqrt{\frac{4 - 4\sqrt{2} + 2}{4 - 2}}$

$\sqrt{\frac{6 - 4\sqrt{2}}{2}}$

$\boxed{\sqrt{3 - 2\sqrt{2}}}$

(10) (a) $\sin x = \frac{5}{13}$, $\sec x > 0$



$* \sin 2x = 2\sin x\cos x$
$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$
$= \boxed{\frac{120}{169}}$

$* \cos 2x = 1 - 2\sin^2 x$
$= 1 - 2\left(\frac{5}{13}\right)^2$
$= 1 - 2\left(\frac{25}{169}\right)$
$= \frac{169}{169} - \frac{50}{169}$
$= \boxed{\frac{119}{169}}$

$* \tan 2x = \frac{\sin 2x}{\cos 2x}$
$= \frac{120/169}{119/169}$
$= \boxed{\frac{120}{119}}$

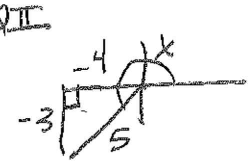
(b) $\cot x = \frac{2}{3}$, $\sin x > 0$

$* \sin 2x = 2\sin x\cos x$
$= 2\left(\frac{3}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{13}}\right)$
$= \boxed{\frac{12}{13}}$

$* \cos 2x = 2\cos^2 x - 1$
$= 2\left(\frac{2}{\sqrt{13}}\right)^2 - 1$
$= \frac{8}{13} - 1$
$= \boxed{-\frac{5}{13}}$

$* \tan 2x = \frac{\sin 2x}{\cos 2x}$
$= \frac{12/13}{-5/13}$
$= \boxed{-\frac{12}{5}}$

(11) (a) $\cos x = -\frac{4}{5}$, $\csc x < 0$, so $\pi < x < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$, so $\frac{x}{2} \to$ QII



$* \sin\left(\frac{x}{2}\right) = +\sqrt{\frac{1}{2}(1-\cos x)}$
$= \sqrt{\frac{1}{2}\left(1-\left(-\frac{4}{5}\right)\right)}$
$= \sqrt{\frac{1}{2}\left(\frac{9}{5}\right)}$
$= \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$
$= \boxed{\frac{3\sqrt{10}}{10}}$

$* \cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1}{2}(1+\cos x)}$
$= -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)}$
$= -\sqrt{\frac{1}{2}\left(\frac{1}{5}\right)}$
$= -\frac{1}{\sqrt{10}}$
$= \boxed{-\frac{\sqrt{10}}{10}}$

$* \tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$
$= \frac{3\sqrt{10}/10}{-\sqrt{10}/10}$
$= \boxed{-3}$

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(11) (b) $\sec x = \frac{3}{2}$, $\tan x < 0$, so $\frac{3\pi}{2} < x < 2\pi$ and $\frac{3\pi}{4} < \frac{x}{2} < \pi$, so $\frac{x}{2} \to$ QII

$*\sin\left(\frac{x}{2}\right)$

$= +\sqrt{\frac{1}{2}(1-\cos x)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{2}{3}\right)}$

$= \sqrt{\frac{1}{6}}$

$= \boxed{\frac{\sqrt{6}}{6}}$

$*\cos\left(\frac{x}{2}\right)$

$= -\sqrt{\frac{1}{2}(1+\cos x)}$

$= -\sqrt{\frac{1}{2}\left(1+\frac{2}{3}\right)}$

$= -\sqrt{\frac{5}{6}} = -\frac{\sqrt{5}}{\sqrt{6}}$

$= \boxed{-\frac{\sqrt{30}}{6}}$

$*\tan\left(\frac{x}{2}\right)$

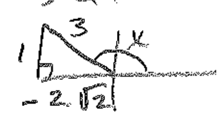
$= \frac{\sin(x/2)}{\cos(x/2)}$

$= \frac{\sqrt{6}/6}{-\sqrt{30}/6}$

$= -\sqrt{\frac{6}{30}} = -\sqrt{\frac{1}{5}}$

$= \boxed{-\frac{\sqrt{5}}{5}}$

(c) $\csc x = 3$, $\cos x < 0$, so $\frac{\pi}{2} < x < \pi$ and $\frac{\pi}{4} < x < \frac{\pi}{2}$, so $\frac{x}{2} \to$ QI



$*\sin\left(\frac{x}{2}\right)$

$= +\sqrt{\frac{1}{2}(1-\cos x)}$

$= \sqrt{\frac{1}{2}\left(1+\frac{2\sqrt{2}}{3}\right)}$

$= \sqrt{\frac{1}{2}\left(\frac{3+2\sqrt{2}}{3}\right)}$

$= \sqrt{\frac{3+2\sqrt{2}}{6}}$

$= \frac{\sqrt{3+2\sqrt{2}}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$

$= \boxed{\frac{\sqrt{18+12\sqrt{2}}}{6}}$

$*\cos\left(\frac{x}{2}\right)$

$= +\sqrt{\frac{1}{2}(1+\cos x)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{2\sqrt{2}}{3}\right)}$

$= \sqrt{\frac{1}{2}\left(\frac{3-2\sqrt{2}}{3}\right)}$

$= \sqrt{\frac{3-2\sqrt{2}}{6}}$

$\frac{\sqrt{3-2\sqrt{2}}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$

$\boxed{\frac{\sqrt{18-12\sqrt{2}}}{6}}$

$*\tan\left(\frac{x}{2}\right)$

$= \frac{\sin(x/2)}{\cos(x/2)}$

$= \frac{\sqrt{18+12\sqrt{2}}/6}{\sqrt{18-12\sqrt{2}}/6}$

$= \frac{\sqrt{18+12\sqrt{2}}}{\sqrt{18-12\sqrt{2}}}\left(\frac{\sqrt{18-12\sqrt{2}}}{\sqrt{18-12\sqrt{2}}}\right)$ *rationalize

$= \frac{\sqrt{(18+12\sqrt{2})(18-12\sqrt{2})}}{18-12\sqrt{2}}$

$= \frac{\sqrt{324-288}}{18-12\sqrt{2}} = \frac{6}{6(3-2\sqrt{2})}$

$= \frac{1}{3-2\sqrt{2}}\left(\frac{3+2\sqrt{2}}{3+2\sqrt{2}}\right)$

$= \frac{3+2\sqrt{2}}{9-8}$

$= \boxed{3+2\sqrt{2}}$

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(12) (a) $\sin 4x \stackrel{?}{=} 2\sin 2x\cos 2x$

$\sin(2(2x))$

$2\sin 2x\cos 2x$ – done

(b) $\cos 6x \stackrel{?}{=} 2\cos^2 3x - 1$

$\cos(2(3x))$

$2\cos^2 3x - 1$ – done

(c) $\sin 3x = \sin x(4\cos^2 x - 1)$

$\sin(2x+x)$

$\sin 2x\cos x + \sin x\cos 2x$

$2\sin x\cos^2 x + \sin x(2\cos^2 x - 1)$

$\sin x(2\cos^2 x + 2\cos^2 x - 1)$

$\sin x(4\cos^2 x - 1)$ – done

(d) $\cos 4x \stackrel{?}{=} 1 - 8\sin^2 x\cos^2 x$

$\cos(2(2x))$

$1 - 2\sin^2 2x$

$1 - 2(\sin 2x)^2$

$1 - 2(2\sin x\cos x)^2$

$1 - 2(4\sin^2 x\cos^2 x)$

$1 - 8\sin^2 x\cos^2 x$ – done

(e) $\sin^4 x \stackrel{?}{=} \frac{1}{8}(3 - 4\cos 2x + \cos 4x)$

$(\sin^2 x)^2$

$\left(\frac{1}{2}(1-\cos 2x)\right)^2$

$\frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$

$\frac{1}{4}\left(1 - 2\cos 2x + \left(\frac{1}{2}(1+\cos 4x)\right)\right)$

$\frac{1}{4}\left(1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right)$

$\frac{1}{4}\left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right)$

$\frac{1}{8}(3 - 4\cos 2x + \cos 4x)$ – done

(f) $\sin^3(2x) \stackrel{?}{=} \left(\frac{\sin(-2x)}{2}\right)(\cos(-4x) - 1)$

$\sin(2x)\cdot\sin^2(2x)$

$\sin 2x\left(\frac{1}{2}(1-\cos 4x)\right)$

$\frac{\sin 2x}{2}(1 - \cos 4x)$

$-\frac{\sin 2x}{2}(\cos 4x - 1)$

$\left(\frac{\sin 2x}{2}\right)(-1)(\cos 4x - 1)$

$\frac{\sin 2x}{2}(1 - \cos 4x)$

– done!