

Name _____ Date _____ Period _____

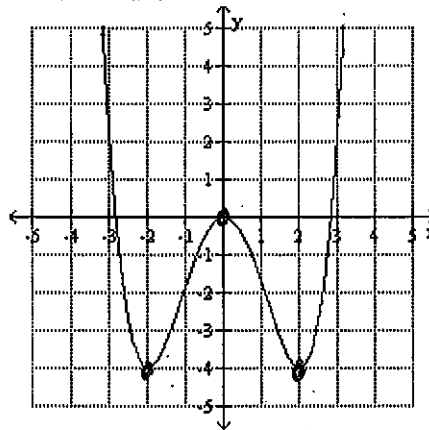
Worksheet 2.3—Other Properties of Functions

Show all work on a separate sheet of paper. Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

- Which of the following scenarios of functions is continuous?
 - (A) The number of students enrolled at New Braunfels High School at any given time
 - (B) The outdoor temperature as a function of time
 - (C) The cost of a U.S. postage stamp as a function of the weight of the letter
 - (D) The price of a stock as a function of time
 - (E) Number of drinks sold at an outdoor event as a function of the outdoor temperature
- Which of the following functions is decreasing? (**Calculator OK—Ha!**)
 - (A) The outdoor temperature as a function of time
 - (B) The Dow Jones Industrial average as a function of time
 - (C) Air pressure in the Earth’s atmosphere as a function of altitude
 - (D) World population since 1900 as a function of time
 - (E) Water pressure in the ocean as a function of depth

Use the graph below to answer questions 3 and 4



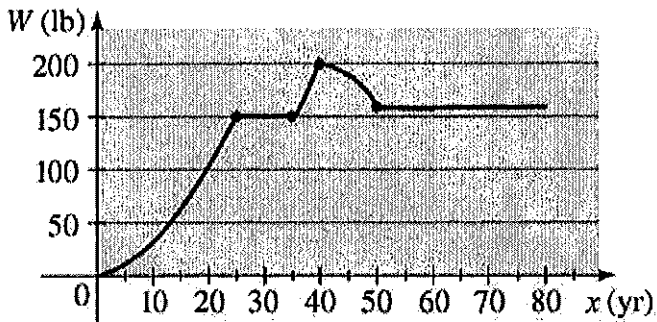
- Estimate the local extrema.
 - (A) Local Max of 0; Local Min of -4
 - (B) No Local Max; Local Min of -4
 - (C) No Local Extrema
 - (D) Local Max of 0; Local Mins of -2, 2
 - (E) Local Max of ∞; Local Mins of -2, 2
- Find the open intervals of increasing/decreasing/constant
 - (A) Inc: (0, ∞); Dec: (-4, 0)
 - (B) Inc: (-2, 2); Dec: (-∞, -2) ∪ (2, ∞)
 - (C) Inc: (-2, 0) ∪ (2, ∞); Dec: (-∞, -2) ∪ (0, 2)
 - (D) Inc: (0, ∞); Dec: (-∞, 0)
 - (E) Inc: (-∞, ∞)

5. $\lim_{x \rightarrow -\infty} \frac{-2x^3 + 3x - 4}{x^2 - 4} =$ (A) 1 (B) -2 (C) 0 (D) ∞ (E) -∞

$\frac{-2(-\infty)^3}{(-\infty)^2} = \frac{+\infty}{+\infty}$

6. Which of the following is NOT true about the function $f(x) = \frac{3x^2 - 27}{x^2 - 25}$
- (A) It is an even function (B) It has a HA at $x=3$ (C) It has VA's at $x = -5, x = 5$
- (D) It has x -intercepts at $x = -3, x = 3$ (E) It has a Local Max at $(0, \frac{27}{25})$

7. Grandpa George is still 80 years old and is still reminiscing about how his wait has fluctuated over his years. He envisions the following piece-wise graph in his head. What is Grandpa's average yearly weight gain from the day of his birth to his 30th birthday (in pounds per year)

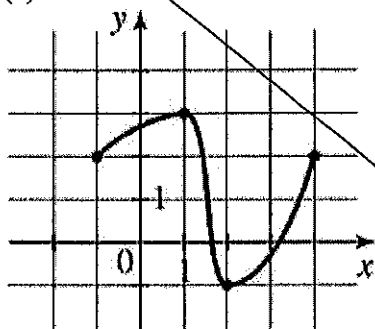


- (A) 200 (B) 150 (C) 100 (D) 50 (E) 5

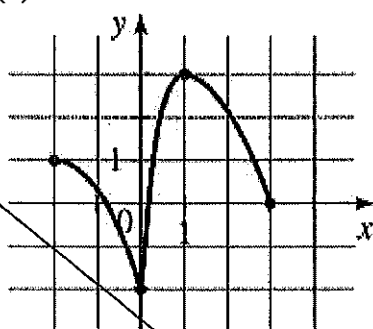
II. Short Answer

8. For each graph below, determine and label (i) domain and range, (ii) the open intervals of increasing/decreasing/constant, and (iii) coordinates of relative extrema. Each graph shows the function in its entirety.

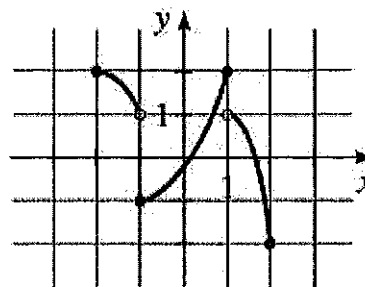
(a)



(b)



(c) (this one has tricky extrema)



9. (Calculator OK) To three decimals, for each of the following, carefully and deliberately, find and label each of the following: (i) Domain and Range, (ii) Coordinates of any Local Extrema, and (iii) Open intervals of increasing/decreasing/constant. Don't list any symmetries! None of these will have any!!

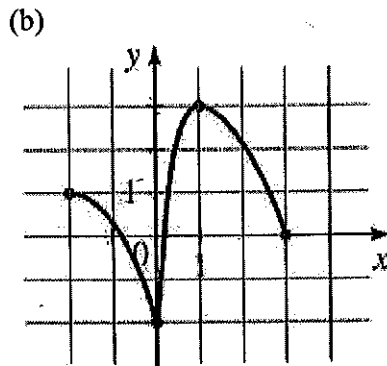
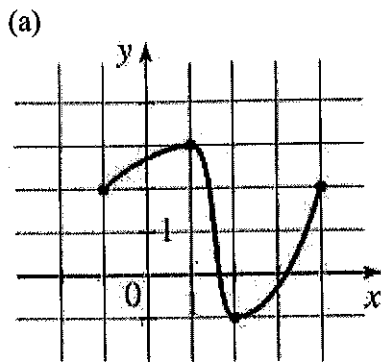
(a) $f(x) = x^3 + 2x^2 - x - 2$ (b) $g(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ (c) $f(x) = \frac{-3x}{2x-3}$

10. For the following functions, find the average rate of change on the indicated interval. Be sure to show the difference quotient.

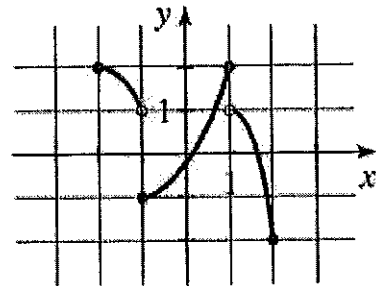
(a) $f(x) = 3x - 2, x \in [2, 3]$ (b) $f(z) = 1 - 3z^2, -2 \leq z \leq 0$ (c) $g(t) = \frac{2}{t+1}, t \in [0, 5]$

(d) $f(x) = x^2 + x - 2, c \leq x \leq c+h$

8. For each graph below, determine and label (i) domain and range, (ii) the open intervals of increasing/decreasing/constant, and (iii) coordinates of relative extrema. Each graph shows the function in its entirety.



(c) (this one has tricky extrema)



a) i) $D: [-1, 4]$
 $R: [-1, 3]$

b) i) $D: [-2, 3]$
 $R: [-2, 3]$

c) i) $D: [-2, 2]$
 $R: [-2, 2]$

ii) $Inc: (-1, 1) \cup (2, 4)$ $Dec: (1, 2)$

ii) $Inc: (-1, 1)$
 $Dec: (-2, -1) \cup (1, 2)$

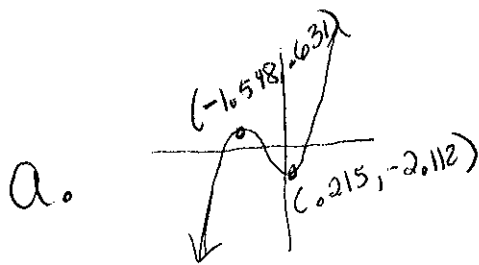
iii) Rel max of 3 @ 1
 Rel min of -1 @ 2

iii) Rel max of 3 @ 1
 Rel min of -2 @ 0

iii) ~~none~~
 Rel max of 2 @ 1
 Rel min of -1 @ -1

9. (Calculator OK) To three decimals, for each of the following, carefully and deliberately, find and label each of the following: (i) Domain and Range, (ii) Coordinates of any Local Extrema, and (iii) Open intervals of increasing/decreasing/constant. Don't list any symmetries! None of these will have any!!

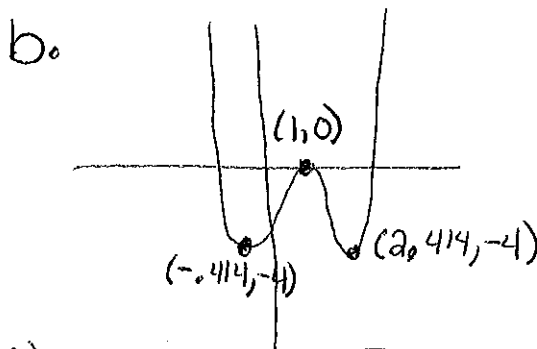
(a) $f(x) = x^3 + 2x^2 - x - 2$ (b) $g(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ (c) $f(x) = \frac{-3x}{2x-3}$



i) Domain \mathbb{R}
Range \mathbb{R}

ii) Rel max of 0.631 @ -1.548
Rel min of -2.112 @ 0.215

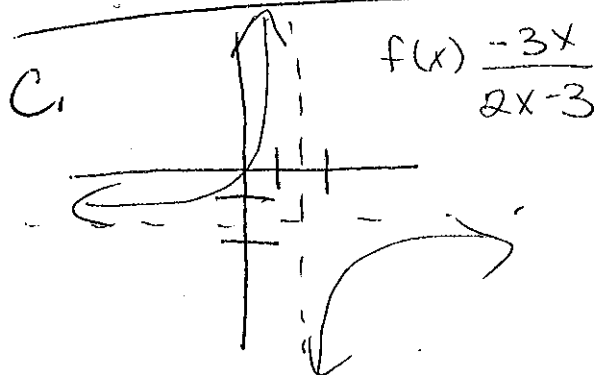
iii) Inc: $(-\infty, -1.548) \cup (0.215, \infty)$
Dec: $(-1.548, 0.215)$



i) Domain: \mathbb{R}
Range: $[-4, \infty)$

ii) Rel max of 0 at 1
Rel min of -4 at -0.414
Rel min of -4 at 2.414

iii) Inc $(-0.414, 1) \cup (2.414, \infty)$
Dec: $(-\infty, -0.414) \cup (1, 2.414)$



$$\begin{aligned} 2x-3 &\neq 0 \\ 2x &\neq 3 \\ x &\neq \frac{3}{2} \end{aligned}$$

i) Domain $\{x \mid x \neq \frac{3}{2}\}$
Range $\{y \mid y \neq -\frac{3}{2}\}$

ii) no rel min or max

iii) inc. $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$
Monotonic Inc.

10. For the following functions, find the average rate of change on the indicated interval. Be sure to show the difference quotient.

(a) $f(x) = 3x - 2, x \in [2, 3]$ (b) $f(z) = 1 - 3z^2, -2 \leq z \leq 0$ (c) $g(t) = \frac{2}{t+1}, t \in [0, 5]$

(d) $f(x) = x^2 + x - 2, c \leq x \leq c+h$

$$a) = \frac{f(3) - f(2)}{3 - 2}$$

$$= \frac{7 - 4}{3 - 2}$$

$$= \frac{3}{1}$$

$$b) = \frac{f(0) - f(-2)}{0 - (-2)}$$

$$= \frac{1 - 12}{2}$$

$$= \frac{-11}{2}$$

$$= -5.5$$

$$c) = \frac{f(5) - f(0)}{5 - 0}$$

$$= \frac{\frac{1}{6} - 2}{5}$$

$$= -\frac{11}{30}$$

$$\frac{\frac{1}{6} - 2}{5} = \frac{-\frac{11}{6}}{5} = -\frac{11}{30}$$

$$d) = \frac{f(c+h) - f(c)}{(c+h) - c}$$

$$= \frac{(c+h)^2 + (c+h) - 2 - (c^2 + c - 2)}{h}$$

$$= \frac{c^2 + 2ch + h^2 + c + h - 2 - c^2 - c + 2}{h}$$

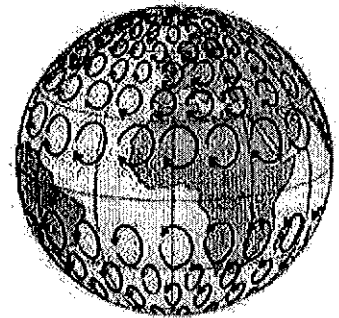
$$= \frac{2ch + h^2 + h}{h} = \frac{h(2c + h + 1)}{h}$$

$$= 2c + h + 1$$

11. A student being chased by a math teacher trying to assign him math homework is running around the track out by the football field. The math teacher records the blazing fast student's time every half lap (200 m), obtaining the data below.

| Time (s) | Distance (m) |
|----------|--------------|
| 32 | 200 |
| 68 | 400 |
| 108 | 600 |
| 152 | 800 |
| 203 | 1000 |
| 263 | 1200 |
| 335 | 1400 |
| 412 | 1600 |

- (a) What was the student's average speed between 68 s and 152 s?
- (b) What was the student's average speed between 263 s and 412 s?
- (c) Calculate the student's average speed for each lap (not half lap). Is the student speeding up, slowing down, or neither. Show the calculations that lead to your answer.
- (d) **Genius question:** Assuming the student is running around the track counterclockwise and the astute math teacher runs clockwise to catch him, when they are 40 m from each other, how does the Coriolis Effect affect the teacher's aim if he were to throw the homework assignment to his student? Assume these shenanigans are taking place here in the Northern Hemisphere.



$$\begin{aligned}
 a) &= \frac{800 - 400}{152 - 68} \\
 &= \frac{400}{84} \quad \begin{array}{l} 400 \text{ meters per } 84 \text{ seconds} \\ 100 \text{ meters per } 21 \text{ seconds} \\ \approx 4.762 \text{ meters per second.} \end{array}
 \end{aligned}$$

$$\begin{aligned}
 b) &= \frac{1600 - 1200}{412 - 263} \\
 &= \frac{400}{149} \\
 &\approx 2.685 \text{ meters per seconds}
 \end{aligned}$$

c) lap 1 68 seconds
 lap 2 152 - 68 = 84 seconds
 lap 3 263 - 152 111 seconds
 lap 4 412 - 263 149 seconds
 Slowing Down

d) HA!

12. Determine algebraically if each of the following are even, odd, or neither.

(a) $f(x) = 4x^5 - 3x^3 + x - 1$ (b) $g(x) = -6\sqrt[5]{6x}$ (c) $h(j) = \sqrt{2j-1}$

(d) $d(y) = \frac{y^2 - \pi}{3y - \sqrt[3]{2y}}$ (e) $k(v) = \frac{2v-1}{7v^2 - 6v + 4}$ (f) $s(w) = \frac{w^{4/5}}{w^{59} - 16w^{33}}$

a) $f(-x) = 4(-x)^5 - 3(-x)^3 + (-x) - 1$

$f(x) = -4x^5 + 3x^3 - x - 1$

Neither

b) $g(-x) = -6\sqrt[5]{6(-x)}$

$g(x) = 6\sqrt[5]{6x}$

ODD

c. $h(-j) = \sqrt{2(-j)-1}$

neither

d. $d(y) = \frac{y^2 - \pi}{3y - \sqrt[3]{2y}}$

$d(-y) = \frac{(-y)^2 - \pi}{3(-y) - \sqrt[3]{2(-y)}}$

$d(-y) = \frac{y^2 - \pi}{-3y + \sqrt[3]{2y}}$ $\frac{\text{Even}}{\text{ODD}} = \text{ODD}$

e) $k(-v) = \frac{2(-v)-1}{7(-v)^2 - 6(-v) + 4} = \frac{\text{neither}}{\text{neither}} = \text{neither}$

f) $s(-w) = \frac{(-w)^{4/5}}{(-w)^{59} - 16(w)^{33}} =$

$\frac{w^{4/5}}{-w^{59} + 16w^{33}}$ $\frac{\text{even}}{\text{ODD}} = \text{ODD}$

13. For each of the following, find the domain, end behavior, x-intercepts, y-intercepts, any symmetry, and find and label all discontinuities. Use your collected information to sketch a graph.

$$(a) f(x) = \frac{-8x^2 - 10x - 3}{2x^2 + x}$$

$$f(x) = \frac{-(8x^2 + 10x + 3)}{2x^2 + x}$$

$$f(x) = \frac{-(2x+1)(4x+3)}{x(2x+1)}$$

Domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)$

$$\lim_{x \rightarrow \infty} f(x) = -4$$

$$\lim_{x \rightarrow -\infty} f(x) = -4$$

VA $x=0$

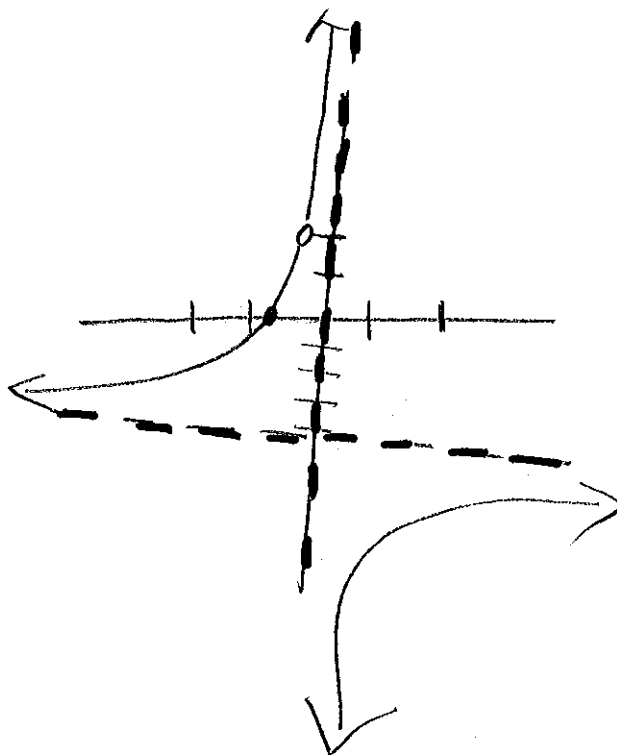
hole $(-\frac{1}{2}, 2)$

HA $y=-4$

$$\begin{aligned} \text{x-int @ } 4x+3 &= 0 \\ x &= -\frac{3}{4} \end{aligned}$$

$$(-\frac{3}{4}, 0)$$

y-int @ $(0, y)$ $x \neq 0$ so no y-int



$$(b) g(t) = \frac{3}{t^2 - 25}$$

$$g(t) = \frac{3}{(t+5)(t-5)}$$

$$t+5 \neq 0 \quad t-5 \neq 0$$
$$t \neq -5 \quad t \neq 5$$

$$\text{Domain: } (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$$

$$\lim_{t \rightarrow \infty} g(t) = 0$$

$$\lim_{t \rightarrow -\infty} g(t) = 0$$

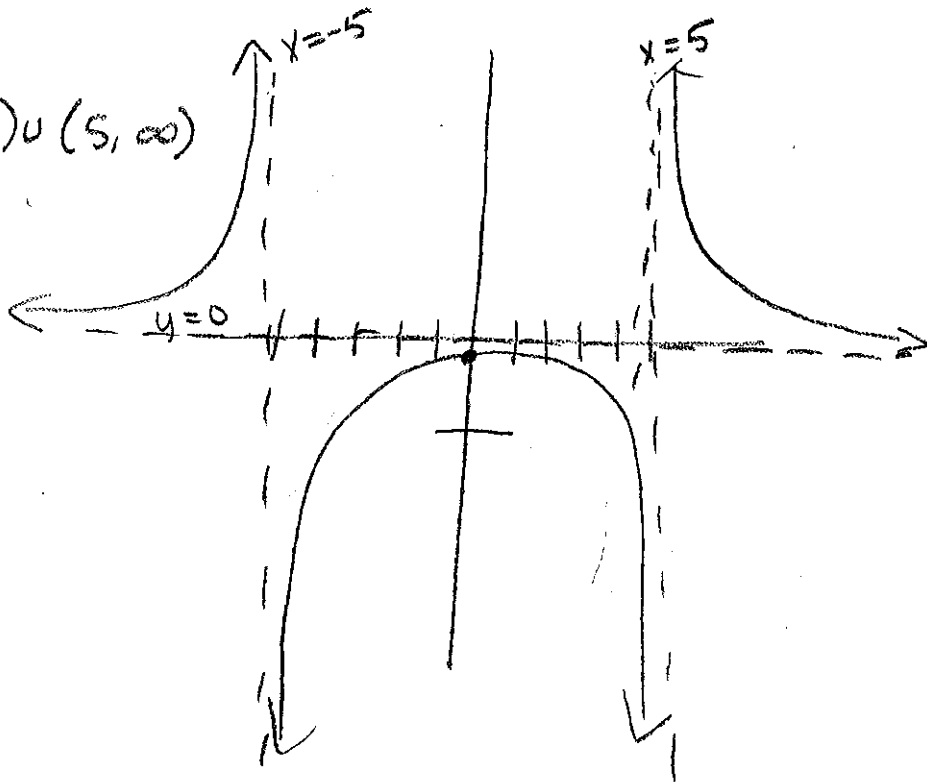
VA at $x = -5, x = 5$

HA at $y = 0$

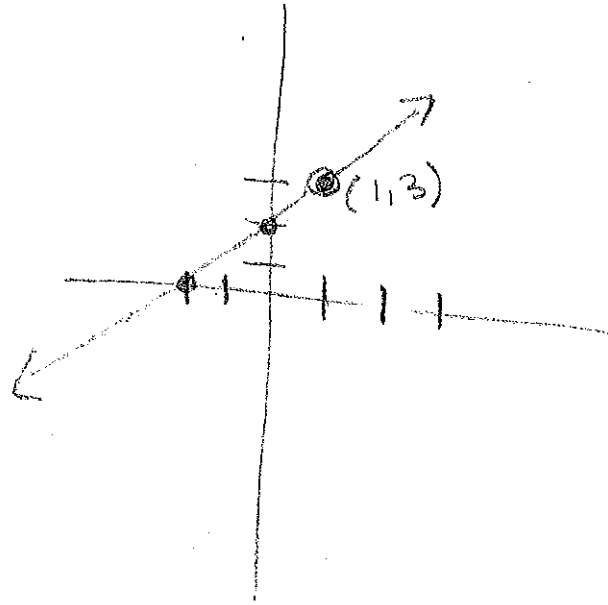
x-int (none)

y-int $(0, -\frac{3}{25})$

Because the function is even
the y axis is the axis of symmetry



$$(c) p(x) = \begin{cases} \frac{x^2 + x - 2}{x - 1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$$



$$p(x) = \frac{(x-1)(x+2)}{x-1}, \quad x \neq 1$$

Domain $(-\infty, 1) \cup (1, \infty)$

$$\lim_{x \rightarrow \infty} p(x) = \infty$$

$$\lim_{x \rightarrow -\infty} p(x) = -\infty$$

x-int $(-2, 0)$

y-int $(0, 2)$

So $p(x) = x + 2$
and is continuous for all $x \in \mathbb{R}$