Name______________________________ Date____________________ Period_____

Worksheet 2.4—Parent Functions & Transformations

Show all work on a separate sheet of paper. Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

1. Give a function *f*, which of the following represents a horizontal stretch by a factor of 3?

   (A) $y = f(3x)$ (B) $y = f\left(\frac{1}{3}x\right)$ (C) $y = 3f(x)$ (D) $y = \frac{1}{3}f(x)$ (E) $y = f(x) + 3$

2. Give a function *f*, which of the following represents a vertical compression by a factor of 3?

   (A) $y = f(3x)$ (B) $y = f\left(\frac{1}{3}x\right)$ (C) $y = 3f(x)$ (D) $y = \frac{1}{3}f(x)$ (E) $y = f(x) + 3$

3. Give a function *f*, which of the following represents a horizontal shift 3 units right?

   (A) $y = f(x-3)$ (B) $y = f(x+3)$ (C) $y = 3 + f(x)$ (D) $y = f(x) - 3$ (E) $y = f(3x)$

4. Give a function *f*, which of the following represents a vertical shift 4 units up FOLLOWED BY a reflection across the *x*-axis??

   (A) $y = -f(x) - 4$ (B) $y = -f(x) + 4$ (C) $y = f(4-x)$ (D) $y = f(x-4)$ (E) $y = -f(x-4)$

5. If $f(x) = 2 + \ln\left(3x - \frac{\pi}{2}\right)$, then compared to the parent function $y = \ln x$, the graph of *f* is shifted

   (A) $\pi/2$ units right (B) $\pi/6$ units right (C) 2 units left (D) 3 units left (E) $\pi/2$ units left

6. The average rate of change for $f(x) = 1 + \sqrt{x}$ on the interval $[1, 4]$ is $\longrightarrow$ $\dfrac{f(4) - f(1)}{4-1}$

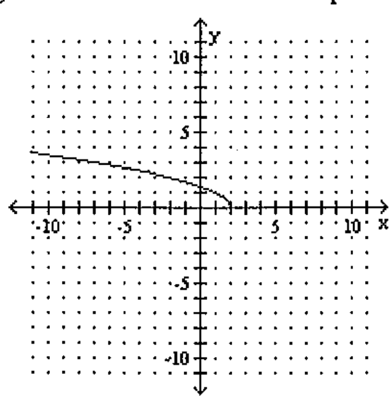
   (A) 1/3 (B) 1/2 (C) 0 (D) 2/3 (E) 3/2

   $\dfrac{1+\sqrt{4} - (1+\sqrt{1})}{4-1}$

   $\dfrac{1+2-(1+1)}{3}$

   $\dfrac{3-2}{3} = \dfrac{1}{3}$

7. The graph of a function $f(x)$ is given below. What is the equation of this graph?



(A) $f(x) = \sqrt{-x} + 2$ (B) $f(x) = -\sqrt{x} + 2$ (C) $f(x) = -\sqrt{x+2}$

(D) $f(x) = \sqrt{-x-2}$ (E) $f(x) = \sqrt{-x+2}$

$= \sqrt{-(x-2)}$

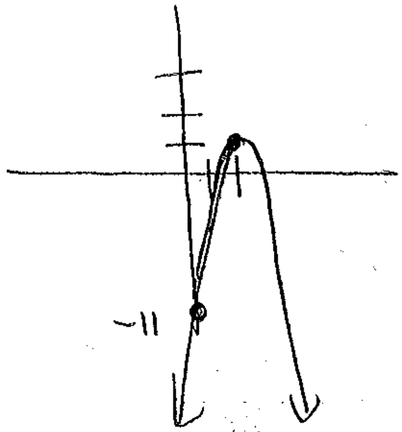
II. Short Answer

8. Find the domain of each function, put each of the following in standard transformation form, then sketch the graph showing important information. Compare your algebraic domain to the domain of your graph.

(a) $f(x) = -3(x-2)^2 + 1$

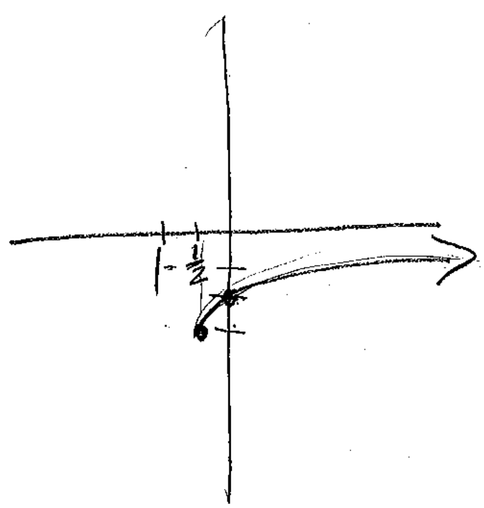
Parent

Domain $\mathbb{R}$



y-int $f(0) = -3(0-2)^2 + 1$
$= -3(-2)^2 + 1$
$= -3(4) + 1$
$= -12 + 1$
$= -11$
$\boxed{(0, -11)}$

(b) $f(x) = \frac{1}{2}\sqrt{8x+4} - 3$



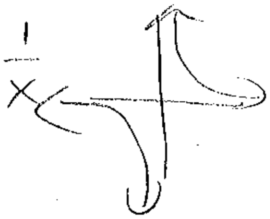
Domain $8x + 4 \geq 0$
$\frac{8x}{8} \geq \frac{-4}{8}$
$x \geq -\frac{1}{2}$

$D_x: \{x \mid x \geq -\frac{1}{2}\}$

$f(x) = \frac{1}{2}\sqrt{8(x+\frac{1}{2})} - 3$

$f(0) = \frac{1}{2}\sqrt{8(0+\frac{1}{2})} - 3$
$= \frac{1}{2}\sqrt{8(\frac{1}{2})} - 3$
$\frac{1}{2}\sqrt{4} - 3$
$\frac{1}{2}(2) - 3 = -2$ $\boxed{(0, -2)}$

parent $y = \frac{1}{x}$



(c) $f(x) = 2 - \frac{3}{2x-5}$

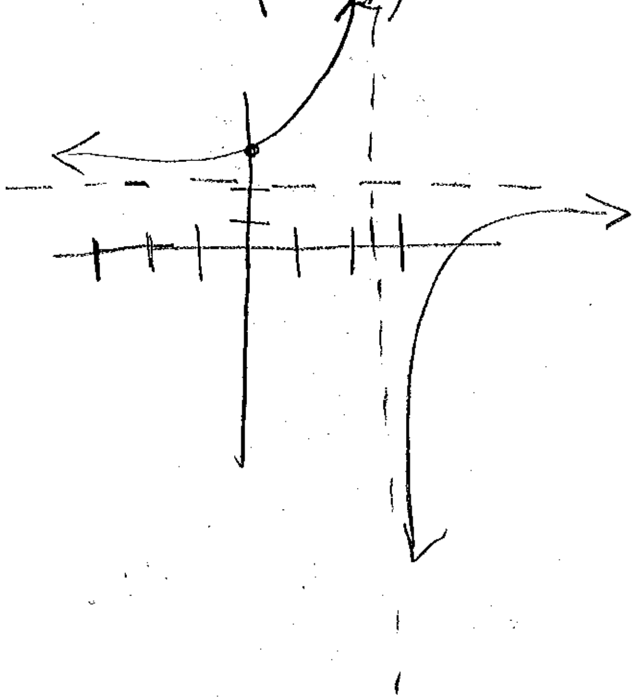
Domain $2x - 5 \neq 0$

$2x \neq 5$

$x \neq \frac{5}{2}$

$\left\{ x \mid x \neq \frac{5}{2} \right\}$

$f(x) = -3\left(\frac{1}{2(x-\frac{5}{2})}\right) + 2$



y-int $y = -3\left(\frac{1}{2(0-\frac{5}{2})}\right) + 2$

$y = -3\left(\frac{1}{2(-\frac{5}{2})}\right) + 2$

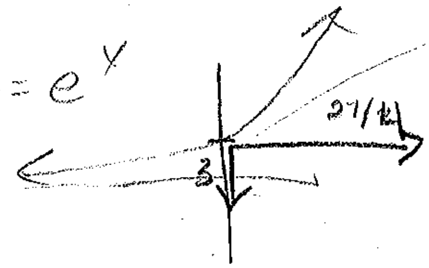
$y = -3\left(\frac{1}{-5}\right) + 2$

$y = \frac{3}{5} + 2$

$y = \frac{13}{5}$ $\left(0, \frac{13}{5}\right)$

(d) $f(x)=\frac{3}{2}e^{\frac{2}{3}x-\frac{9}{2}}-3$

$f(x)=e^x$



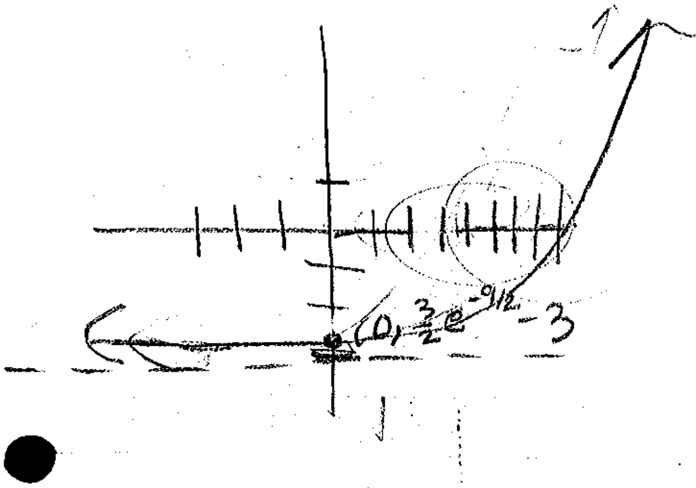
Domain $\mathbb{R}$

$f(x)=\frac{3}{2}e^{\frac{2}{3}(x-\frac{27}{4})}-3$

$f(0)=\frac{3}{2}e^{\frac{2}{3}(0-\frac{27}{4})}-3$

$=\frac{3}{2}e^{-9/2}-3$

$-\frac{9}{2}\cdot\frac{3}{2}=$
$-\frac{27}{4}$



(e) $f(x)=\frac{2x-3}{4x-1}$

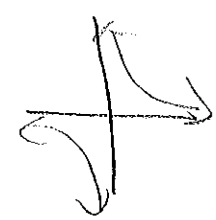
$$\begin{array}{r} \frac{1}{2}+\frac{-\frac{5}{2}}{4x-1} \\ 4x-1\,\overline{)\,2x-3} \\ -2x+\frac{1}{2} \\ \hline -\frac{5}{2} \end{array}$$

y-int

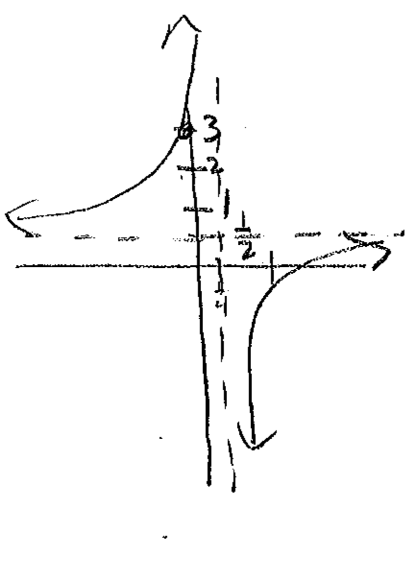
$f(0)=\frac{2(0)-3}{4(0)-1}$

$f(0)=3$

$f(x)=\frac{1}{x}$



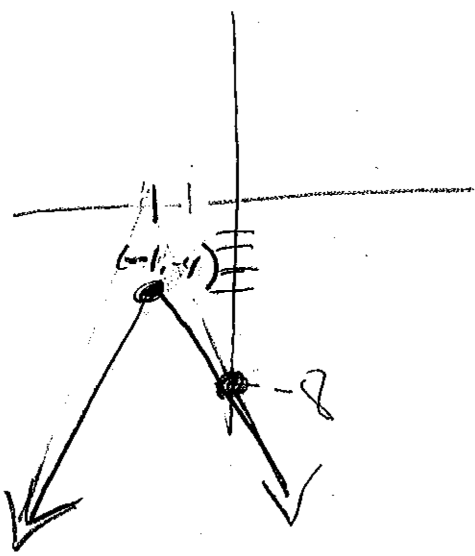
$f(x)=-\frac{5}{2}\left(\frac{1}{4(x-\frac{1}{4})}\right)+\frac{1}{2}$



(f) $f(x) = -2 - 2|-2x - 2| - 2$

Domain $\mathbb{R}$ $\{x \mid x \in \mathbb{R}\}$

$f(x) = -2|-2(x+1)| - 4$



y int

$f(0) = -2|-2(0+1)| - 4$

$= -2|-2| - 4$

$= -2(2) - 4$

$= -4 - 4$

$= -8$

9. If $g(x) = 2f(x)$ is a transformation of a function $y = f(x)$, by algebraically manipulating each function, describe TWO different ways to obtain the graph of g from the graph of f.

(a) $f(x) = x$ (b) $f(x) = x^2$ (c) $f(x) = x^3$ (d) $f(x) = \sqrt{x}$ (e) $f(x) = x^{-1}$

(f) $f(x) = |x|$ (g) $f(x) = \ln x$ (h) $f(x) = e^x$

a. $f(x) = x$

$g(x) = 2(x)$ vert. stretch bfo 2

OR

$g(x) = (2x)$ vert. comp bfo 2

b. $f(x) = x^2$

$g(x) = 2x^2$ vert stretch bfo 2

$g(x) = (\sqrt{2}x)^2$ horz comp bfo $\sqrt{2}$

c. $f(x) = x^3$

$g(x) = 2x^3$ V stretch bfo 2

$g(x) = (2^{\frac{1}{3}}x^3)^3$ H Comp bfo $2^{\frac{1}{3}}$

d. $f(x) = \sqrt{x}$

$g(x) = 2\sqrt{x}$ V stretch bfo 2

$g(x) = \sqrt{4x}$ H comp bfo 4

e. $f(x) = x^{-1}$

$g(x) = 2x^{-1}$ V stretch bfo 2

$g(x) = (\frac{1}{2}x)^{-1}$ H stretch bfo 2

f. $f(x) = |x|$

$g(x) = 2|x|$ V stretch bfo 2

$g(x) = |2x|$ H comp bfo 2

g. $f(x) = \ln x$

$g(x) = 2\ln x$ no equiv. standard trans

$g(x) = \ln x^2$

h. $f(x) = e^x$

$g(x) = 2e^x$ - Vert. stretch bfo 2

$g(x) = (2^{\frac{1}{x}}e)^x = e^{x+\ln 2}$

Horz shift left ln2

This probeem is from an older version of the WS

9. If $g(x) = f(ex)$ is a transformation of a function $y = f(x)$, by algebraically manipulating each function, describe TWO different ways, if possible, to obtain the graph of g from the graph of f by a standard transformation. Note: $e \approx 2.718$.

(a) $f(x) = x$ (b) $f(x) = x^2$ (c) $f(x) = x^3$ (d) $f(x) = \sqrt{x}$ (e) $f(x) = x^{-1}$

(f) $f(x) = |x|$ (g) $f(x) = \ln x$ (h) $f(x) = e^x$

a) $f(x) = x$

$g(x) = (ex)$ $\boxed{\text{HC bfo } e}$

$g(x) = ex$ $\boxed{\text{VS bfo } e}$

b) $f(x) = x^2$

$g(x) = (ex)^2$ $\boxed{\text{HC bfo } e}$

$g(x) = e^2x^2$ $\boxed{\text{VS bfo } e^2}$

c) $f(x) = x^3$

$g(x) = (ex)^3$ $\boxed{\text{HC bfo } e}$

$g(x) = e^3x^3$ $\boxed{\text{VS bfo } e^3}$

d) $f(x) = \sqrt{x}$

$g(x) = \sqrt{ex}$ $\boxed{\text{HC bfo } e}$

$g(x) = e^{1/2}\sqrt{x}$ $\boxed{\text{VS bfo } e^{1/2}}$

e) $f(x) = x^{-1}$

$g(x) = (ex)^{-1}$ $\boxed{\text{HC bfo } e}$

$g(x) = \frac{1}{e}x^{-1}$ $\boxed{\text{VC bfo } e}$

f) $f(x) = |x|$

$g(x) = |ex|$ $\boxed{\text{HC bfo } e}$

$g(x) = e|x|$ $\boxed{\text{VS bfo } e}$

g) $f(x) = \ln x$

$g(x) = \ln ex$ $\boxed{\text{HC bfo } e}$

$g(x) = \ln x + \ln e$ $\boxed{\text{vertical shift up } \ln e}$

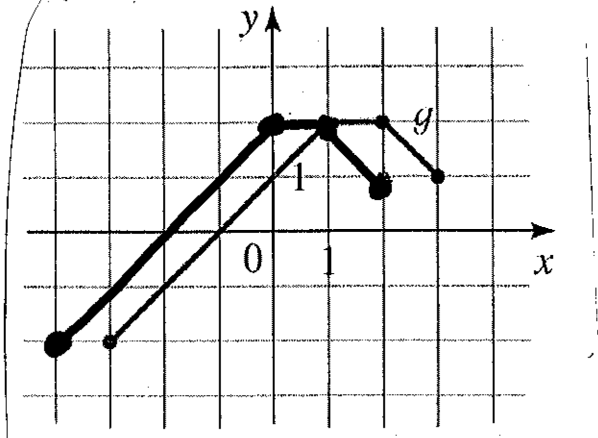
h) $f(x) = e^x$

$g(x) = e^{ex}$

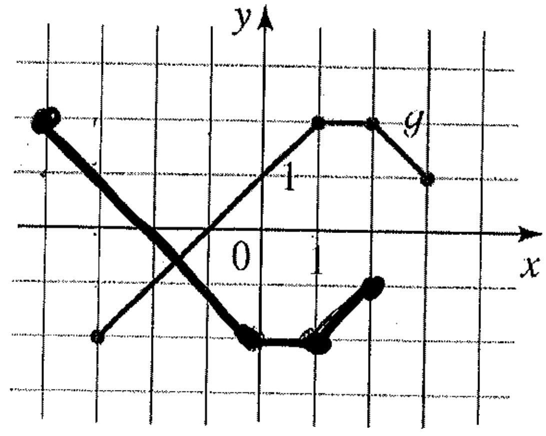
$g(x) = (e^x)^e$ no equiv. standard trans.

• from newer version of WS

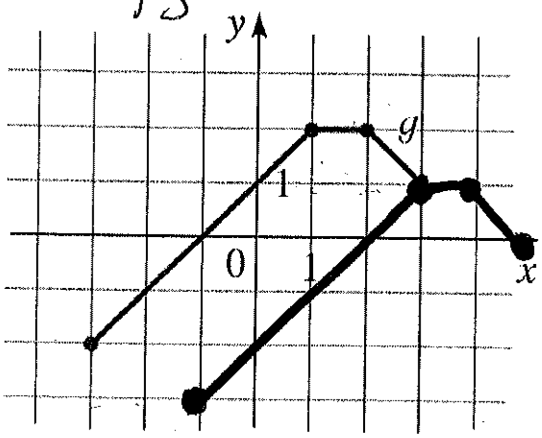
a) $y=h(x+1)$



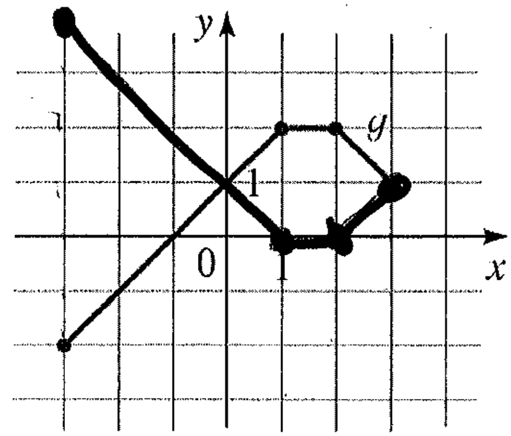
b) $y=-h(x+1)$



c) $y=h(x-2)-1$

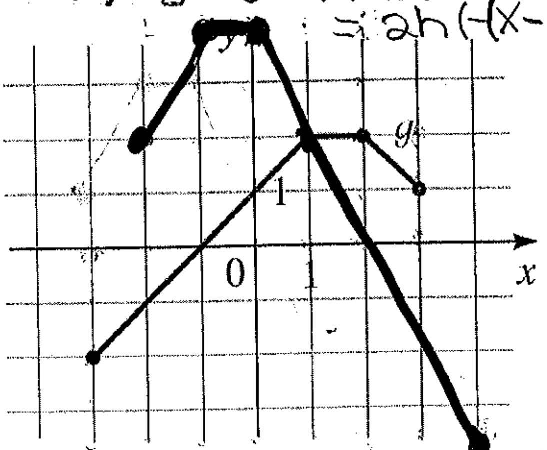


D) $y=-h(x)+2$

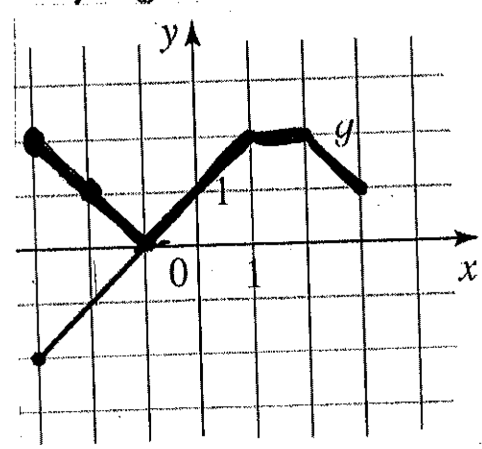


e) $y=2h(1-x)$

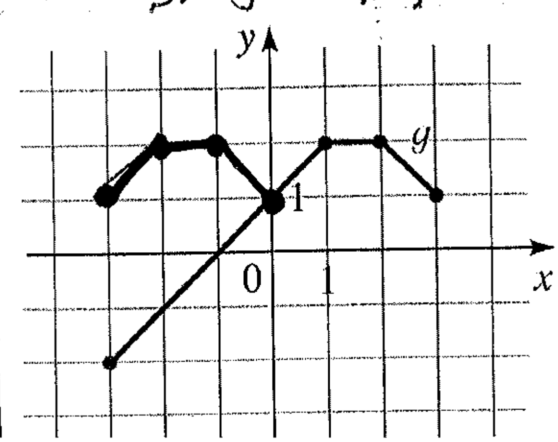
$=2h(-(x-1))$



f) $y=|h(x)|$



g) $y=h(|x|)$



h) $|h(|x|)|$