Name______________________________ Date____________________ Period______

Worksheet 4.1—Exponential and Logistic Functions

Show all work on a separate sheet of paper. All answers must be given as **<u>simplified, exact answers</u>**! No Calculators are permitted unless specified otherwise.

Multiple Choice

1. Which of the following functions is exponential?

   (A) $f(x)=b^2$ (B) $f(x)=x^3$ (C) $f(x)=x^{2/3}$ (D) $f(x)=\sqrt[3]{x}$ (E) $f(x)=8^x$

2. What point do all functions of the form $f(x)=b^x$ have in common?

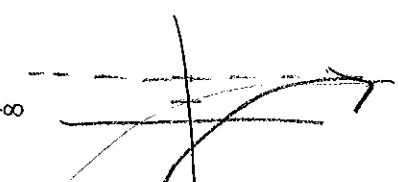
   (A) $(1,1)$ (B) $(1,0)$ (C) $(0,1)$ (D) $(0,0)$ (E) $(-1,-1)$

3. For $x>0$, which of the following is true?

   (A) $3^x>4^x$ (B) $7^x>5^x$ (C) $\left(\frac{1}{6}\right)^x>\left(\frac{1}{2}\right)^x$ (D) $9^{-x}>8^{-x}$ (E) $0.17^x>0.32^x$

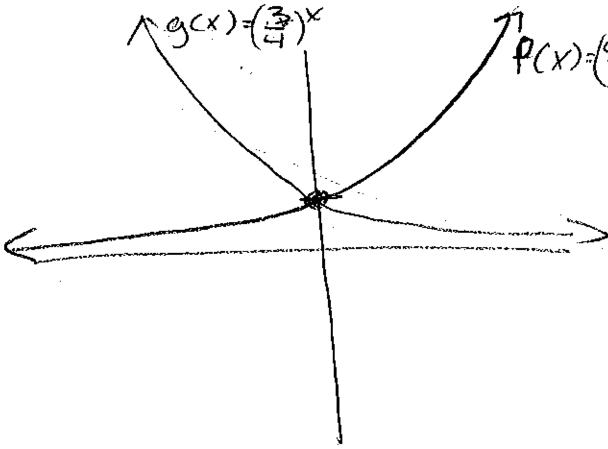
4. If $f(x)=2-3e^{4-7x}$, what is $\lim_{x\to-\infty} f(x)$? $f(x)=-3e^{-7(x-\frac{4}{7})}+2$

   (A) 0 (B) 2 (C) 3 (D) ∞ (E) $-\infty$



5. If $f(x)=2-3e^{4-7x}$, what is $\lim f(x)$?

6. Sketch the functions $f(x)=\left(\frac{4}{3}\right)^x$ and $g(x)=\left(\frac{3}{4}\right)^x$ on the same set of axis. Describe the domain, range, end behavior, find intercepts, and describe how the functions are related.



D_f: $\mathbb{R}$ D_g $\mathbb{R}$

R_f: $(0,\infty)$ R_g $(0,\infty)$

$(0,1)$ $(0,1)$

Reflection across the y-axis.

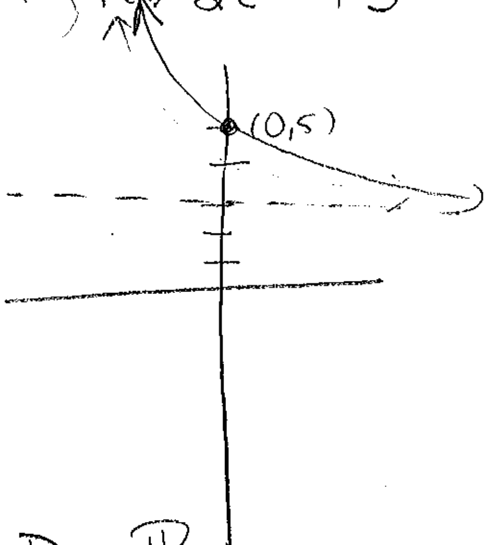
$\lim_{x\to\infty} f(x)=0$ $\lim_{x\to 0} g(x)=0$

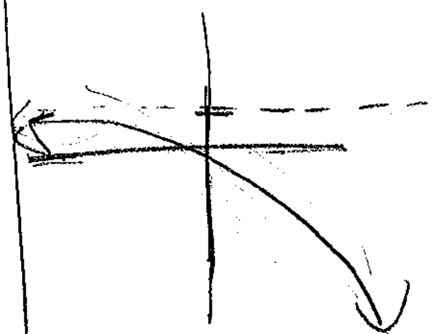
7. Sketch the following functions by using transformations. Describe the domain and range.

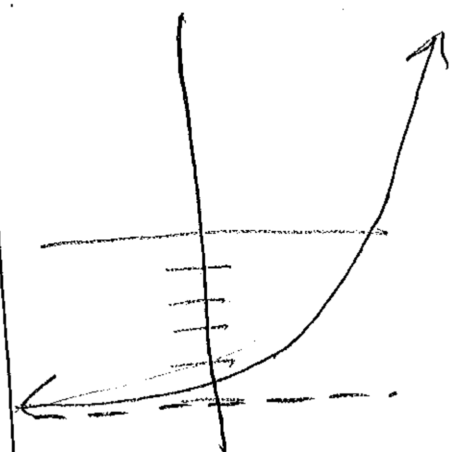
(a) $f(x)=2e^{-\frac{x}{2}}+3$ (b) $g(x)=1-5\cdot\left(\frac{5}{7}\right)^{2-x}$ (c) $h(x)=7\cdot 6^{2x-1}-5$

$g(x)=-5\left(\frac{5}{7}\right)^{-(x-2)}+1$ $h(x)=7\cdot 6^{2(x-\frac{1}{2})}-5$

a) $f(x)=2e^{-\frac{1}{2}x}+3$







D_f: $\mathbb{R}$

R_f: $\{y \mid y>3\}$

D_g: $\mathbb{R}$

R_g: $\{y \mid y<1\}$

D: $\mathbb{R}$

R $\{y \mid y>-5\}$

8. Let $f(x) = 3^x$. If $f(x)$ is vertically compressed by a factor of 9, what would the new equation, $g(x)$, be? The resulting graph can be equivalently obtained by a horizontal shift on $f(x)$. Describe this transformation, and show the work that leads to your answer.

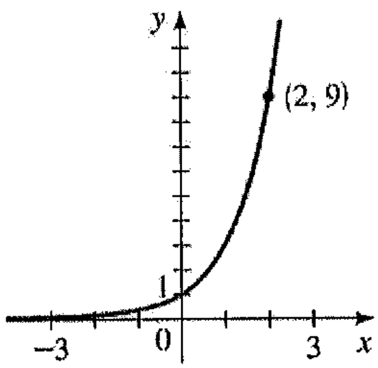
$$\boxed{g(x) = \frac{1}{9}(3)^x}$$

$$g(x) = 3^{-2} \cdot 3^x$$

$$\boxed{g(x) = 3^{x-2}}$$

9. Find the exponential function $f(x) = A \cdot b^x$ whose graph is given or whose points are given. Be sure to read the y-intercept of the graphs to get your second point.

(a)



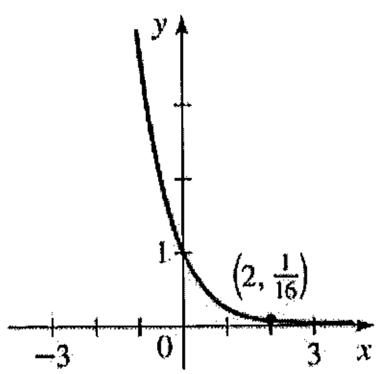
$A = 1$

$9 = 1b^2$

$b = 3$

$$\boxed{f(x) = 3^x}$$

(b)



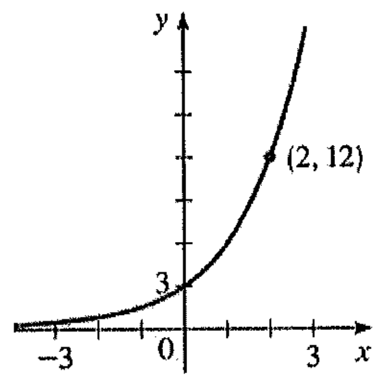
$A = 1$

$\frac{1}{16} = 1b^2$

$b = \frac{1}{4}$

$$\boxed{f(x) = \frac{1}{4}^x}$$

(c)



$A = 3$

$\frac{12}{3} = \frac{3b^2}{3}$

$4 = b^2$

$b = 2$

$$\boxed{f(x) = 3 \cdot 2^x}$$

(c) $(-2,3)$ and $\left(5,\frac{1}{2}\right)$ (d) $(-1,4)$ and $\left(\frac{3}{8},12\right)$

$y = Ab^x$

$3 = Ab^{-2}$

$\frac{1}{2} = Ab^5$

$\left(6^{-1/7}\right) = \left(b^{-7}\right)^{-1/7}$

$b = 6^{-1/7}$

$3 = A\left((6)^{-\frac{1}{7}}\right)^{-2}$

$\frac{3}{6^{2/7}} = \frac{A6^{\frac{2}{7}}}{6^{\frac{2}{7}}}$

$A = \frac{3}{6^{2/7}}$

$y = \left(\frac{3}{6^{2/7}}\right)\left(6^{-\frac{x}{7}}\right) = y = 3\cdot 6^{-\frac{2}{7}}\cdot 6^{-\frac{x}{7}}$ $\boxed{y = 3\cdot 6^{-\frac{(x+2)}{7}}}$

$4 = Ab^{-1}$

$12 = Ab^{\frac{3}{8}}$

Eq 2 ÷ Eq 1

$(3)^{\frac{8}{11}} = \left(b^{\frac{11}{8}}\right)^{\frac{8}{11}}$

$b = 3^{\frac{8}{11}}$

$4 = A\left((3)^{\frac{8}{11}}\right)^{-1}$

$\frac{4}{3^{-\frac{8}{11}}} = \frac{A(3^{-\frac{8}{11}})}{3^{-\frac{8}{11}}}$

$A = \frac{4}{3^{-\frac{8}{11}}}$

$y = \frac{4}{3^{-\frac{8}{11}}}\left(3^{\frac{8}{11}}\right)^x$

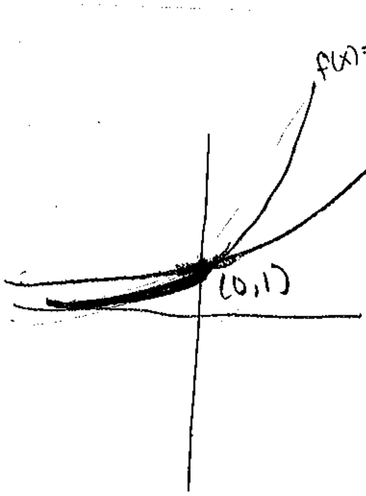
$y = 4\cdot 3^{\frac{8}{11}}\cdot 3^{\frac{8}{11}x}$

$y = 4\cdot 3^{\frac{8(x+1)}{11}}$

10. Solve the following inequalities graphically.

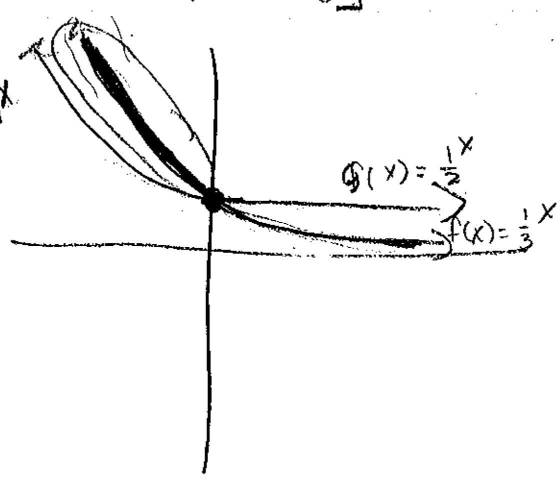
(a) $9^x < 4^x$

$(-\infty, 0)$



(b) $\left(\frac{1}{3}\right)^x \geq \left(\frac{1}{2}\right)^x$

$(-\infty, 0]$



11. Determine if each of the following functions is an exponential growth or decay function, then describe both end behaviors using limit notation.

(a) $f(x)=e^{-2x}$ (b) $f(x)=\frac{1}{\left(\frac{1}{e}\right)^x}$ (c) $f(x)=\left(\frac{1}{0.75}\right)^{-x}$ $\frac{1}{\frac{3}{4}}$

decay growth Decay

12. Determine which of the following, if any, exponential functions are equivalent. Justify your answer.

(a) I. $f(x)=3^{2x+4}$ II. $g(x)=3^{2x}+4$ III. $h(x)=9^{x+2}$ $3^{2(x+2)}$

(b) I. $y_1=4^{3x-2}$ $2^{2(3x-2)}$ II. $y_2=2(2^{3x-2})$ III. $y_3=2^{3x-1}$

13. If $f(x)=10^x$, show that $\frac{f(x+h)-f(x)}{h}=10^x\left(\frac{10^h-1}{h}\right)$ $\frac{10^{x+h}-10^x}{h}=\frac{10^x\cdot 10^h-10^x}{h}$

$=10^x\frac{(10^h-1)}{h}$

14. **Calculator Permitted**

(a) Compare the rates of growth of the functions $f(x)=2^x$ and $g(x)=x^5$ by drawing the graphs of both functions in the following viewing windows. (Turn your xscl and yscl to zero).

(i) $[0,5]$ by $[0,20]$ g(x) is growing faster

(ii) $[0,25]$ by $[0,10^7]$ f(x) is growing faster

(iii) $[0,50]$ by $[0,10^8]$ f(x) is growing faster

(b) Find the solutions of the equation $2^x=x^5$, correct to three decimal places.

(22.44, 5690033.4)

(1.177, 2.261)