11.1 NOTES continued

Matched pairs *t* procedures: One sample, Two treatments

While one-sample inference is fun and valuable, comparative studies are more convincing and, therefore, more common than single sample studies.

One way to do this is through a Match-Pairs design.

Recall that a **matched pairs design** is a design that matches two subjects (two independent populations), or two treatments on the same subject (one population).

We'll focus first on matched pairs involving two different treatments on a single population. **Although there are two treatments, we will use ONE-SAMPLE *t* procedures to the OBSERVED DIFFERENCES.**

The parameter μ will be the MEAN DIFFERENCE in the responses of the two treatments within matched pairs of subjects in the entire population.

Summary:

Matched Pairs T (two treatments) – *Runs like a one-sample t*

Assumptions

- Random samples from a single population
- Given normality of **paired data set** (check distribution shape and normal plot since many sample sizes will be less than 30!!!!!!!)
- Population size at least 10 times sample size,
- Degrees of freedom = $n - 1$.
- Must define μ_d *(mean difference)*.
- Standard deviation unknown, use s_d to approximate.

Confidence Interval

$$\bar{x}_d \pm t^* \frac{s_d}{\sqrt{n}}$$

Test of Significance

$H_0 : \mu_d = 0$ $\quad$ $H_a : \mu_d < 0 \;\; or \;\; \mu_d \neq 0 \;\; or \;\; \mu_d > 0$ Choose one.

$$t = \frac{\bar{x}_d - \mu_d}{s_d / \sqrt{n}}$$

Example 1. Floral Scents and Learning

It is said that listening to Mozart improves students' performance on tests (especially if the Stat professor's name is Mozart). Perhaps pleasant odors have a similar effect (does the Stat professor bathe??) To test this idea, 21 subjects worked a paper-and-pencil maze while wearing a mask. The mask was either unscented or carried a wonderful floral scent. The response variable is their average time on three trials. Each subject worked the maze with both masks, in a random order. The randomization is important because subjects tend to improve their times as they work a maze repeatedly.

SUBJECT	**UNSCENTED** (seconds)	**SCENTED** (seconds)	**Difference** (seconds
1	30.6	37.97	-7.37
2	48.43	51.57	-3.14
3	60.77	56.67	4.1
4	36.07	40.47	-4.40
5	68.47	49	19.47
6	32.43	43.23	-10.8
7	43.7	44.57	-0.87
8	37.1	28.4	8.7
9	31.17	28.23	2.94
10	51.23	68.47	
11	65.4	51.1	
12	58.93	83.5	
13	54.47	38.3	
14	43.53	51.37	
15	37.93	29.33	
16	43.5	54.27	
17	87.7	62.73	
18	53.53	58	
19	64.3	52.4	
20	47.37	53.63	
21	53.67	47	

To analyze the data, we will subtract the scented time from the unscented time for each subject. The 21 separate differences will form a single sample. We generally won't be given these differences, so we'll have to calculate them on our own in from L1 and L2. We'll also have to be careful in defining our two samples so that we can interpret the SIGN of the difference correctly.

Step 1: Let sample 1 be the unscented trial and sample 2 be the scented trial. We'll define μ_d to be sample 1 minus sample 2 or $\mu = \mu_1 - \mu_2$. A positive difference now means that the unscented time was longer and that the subject was faster with the scented mask. A negative difference means the scented mask trial took longer and that the subject was faster without the mask.

Step 2: Our population of interest is subjects working mazes. Our population parameter is the difference between times working a maze with a scented versus and unscented mask. We want to see if the presence of pleasant aromas improve performance, that is, decrease the time it takes to work a maze.

$H_0 : \mu = 0$ Pleasant aromas do not improve one's time in working a maze

$H_a : \mu > 0$ Pleasant aromas improve ones time in working a maze

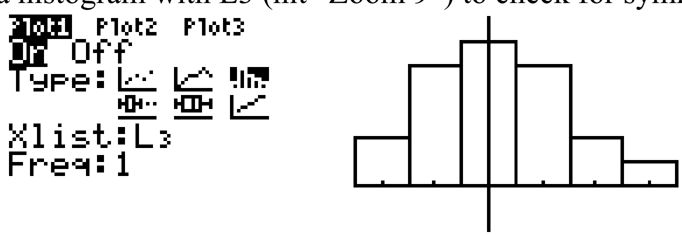
Step 3: choose and state the appropriate inference procedure and verify conditions for the selected procedure:

- Since we do not know the population standard deviation of the differences, we will use a Matched Pairs *t*-Test:
  - The data comes from a randomized, matched pairs experiment. If we are to generalize our results to the entire population, then we'll have to assume that our subjects were chosen from a simple random sample of the entire population.
  - We now have to check to see if the distribution is roughly symmetric and approximately normal, but FIRST, we must calculate our differences. With the unscented data in L1 and the scented in L2, define a difference equation in L2 with quotations around it.

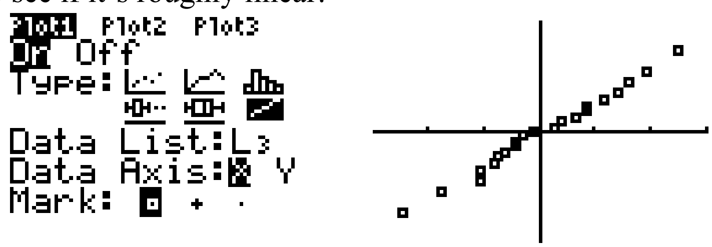
L1	L2	L3
30.6	37.97	-7.37
48.43	51.57	-3.14
60.77	56.67	4.1
36.07	40.47	-4.4
68.47	49	19.47
32.43	43.23	-10.8
43.7	44.57	-.87

L3 ="L1-L2"

Now we'll construct a histogram with L3 (hit "Zoom 9") to check for symmetry



The distribution is single peak and roughly symmetrical. We should also check the normal distribution graph to see if it's roughly linear:

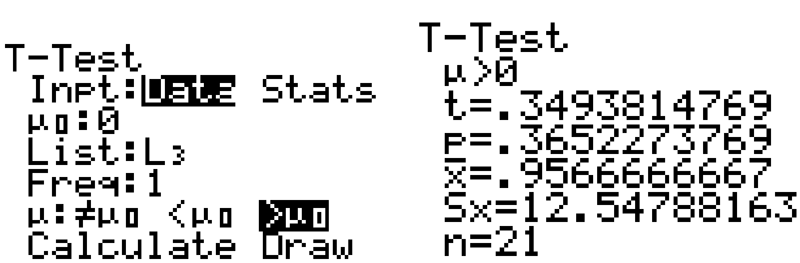


We can now assume with confidence that the distribution of differences is normally distributed.

Step 4: If conditions are met, carry out the inference procedure

Calculate the test statistic. The 21 differences have

$$df = 20, \ \bar{x} = 0.9567, s = 12.5479, \ t = 0.349, \text{ and } \ p = 0.365$$



Step 5: Interpret your results in the context of the problem
The data does NOT support the claim that floral scents improve performance. The average improvement is small, not even a second, over the 50 seconds that the average subject took when wearing the unscented mask. The observed small difference would occur about 37 times out of 100 simply by chance and not as a result of any floral enhancement. Our results aren't even significant at the 36% level, therefore, we fail to reject the null.

A word about using *t*-procedures and normality:

- Except in the case of small samples, the assumption that the data is an SRS from the population of interest is more important than the assumption that the population distribution is normal.
- For a sample size less than 15, use *t* procedures if the data are close to normal. If the data are clearly non-normal or if outliers are present, do not use *t*.
- For a sample size at least 15, the *t* procedures can be used except in the presence of outliers or strong skewness.
- For large samples greater than or equal to roughly 30, the *t* procedures can be used even for clearly skewed distributions (CLT)