Review 10.1 Test Key

① $f(x) = -2x^2 + x + 3$

ⓐ $f'(x) = \lim_{h\to 0} \dfrac{[-2(x+h)^2 + (x+h) + 3] - [-2x^2 + x + 3]}{h}$

$= \lim_{h\to 0} \dfrac{-2x^2 - 4xh - h^2 + x + h + 3 + 2x^2 - x - 3}{h}$

$= \lim_{h\to 0} \dfrac{-4xh - h^2 + h}{h} = \lim_{h\to 0} \dfrac{\cancel{h}(-4x - h + 1)}{\cancel{h}}$

$= \boxed{-4x + 1 = f'(x)}$

ⓑ $f(2) = -2(2^2) + 2 + 3 = -3 \to (2, -3)$

$f'(2) = -4(2) + 1 = -7 = m$

ⓒ $y - y_1 = m(x - x_1)$

$\boxed{y + 3 = -7(x - 2)}$

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② $f(x) = \dfrac{-2}{4-x}$, $f'(x) = \lim_{h\to 0} \dfrac{\dfrac{-2}{4-(x+h)} - \dfrac{-2}{4-x}}{h} \left(\dfrac{(4-x-h)(4-x)}{(4-x-h)(4-x)}\right)$

$f'(x) = \lim_{h\to 0} \dfrac{-2(4-x) - (-2)(4-x-h)}{h[(4-x-h)(4-x)]} = \lim_{h\to 0} \dfrac{-8 + 2x + 8 - 2x - 2h}{h(4-x-h)(4-x)}$

$= \lim_{h\to 0} \dfrac{-2\cancel{h}}{\cancel{h}(4-x-h)(4-x)} = \boxed{\dfrac{-2}{(4-x)^2} = f'(x)}$

ⓑ $f(2) = \dfrac{-2}{4-2} = -1 \to (2, -1)$, $f'(2) = \dfrac{-2}{(4-2)^2} = -\dfrac{1}{2} = m$

ⓒ $\boxed{y + 1 = -\frac{1}{2}(x - 2)}$

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③ $f(x) = \sqrt{6-x}$, $f'(x) = \lim_{h\to 0} \dfrac{\sqrt{6-(x+h)} - \sqrt{6-x}}{h} \cdot \left(\dfrac{\sqrt{6-x-h} + \sqrt{6-x}}{\sqrt{6-x-h} + \sqrt{6-x}}\right)$

$f'(x) = \lim_{h\to 0} \dfrac{6 - x - h - 6 + x}{h(\sqrt{6-x-h} + \sqrt{6-x})} = \lim_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(\sqrt{6-x-h} + \sqrt{6-x})} = \boxed{\dfrac{-1}{2\sqrt{6-x}} = f'(x)}$

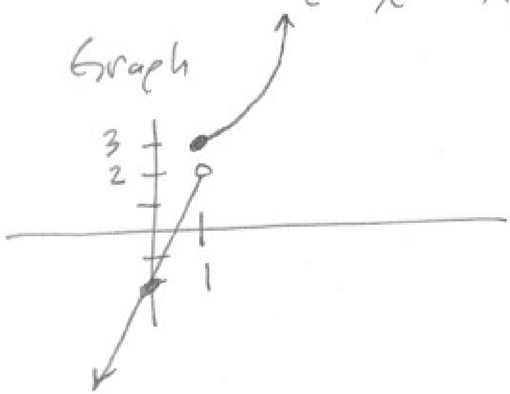
ⓑ $f(2) = \sqrt{6-2} = 2 \to (2, 2)$, $f'(2) = -\dfrac{1}{4} = m$

ⓒ $\boxed{y - 2 = -\frac{1}{4}(x - 2)}$

Review 10.1 Key cont.

(4) $f(x) = \begin{cases} 4x-2, & x<1 \\ x^2+2x, & x \geq 1 \end{cases}$

Graph



(a) $\lim_{x \to 1^-} f(x) = 4(1) - 2 = 2$ (plug into top piece)

(b) $\lim_{x \to 1^+} f(x) = 1^2 + 2(1) = 3$ (plug into bottom piece)

(c) $f(1) = 1^2 + 2(1) = 3$ (since $x \geq 1$)

(d) $f'(1) =$ DNE since a jump discon exists @ $x=1$

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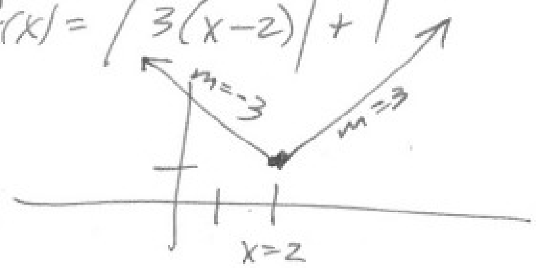
(5) $f(x) = |3x - 6| + 1$

slope $= \pm 3$

(a) $f(x) = |3(x-2)| + 1$



(b) $f(2) = |3(2) - 6| + 1 = 1 \rightarrow (2,1)$

(c) $f'(0) = -3$ (since to left of vertex @ $x=2$)

(d) $f'(5) = 3$ (since to right of vertex @ $x=2$)

(e) $f'(2) =$ DNE (sharp turn)

Bonus on test: Sketch a graph of f from f'