CALCULUS
WORKSHEET 2 ON LIMITS

1. Given that $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} h(x) = 8$, find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to a}\left[f(x)+h(x)\right] =$

(b) $\lim_{x\to a}\left[f(x)\right]^2 =$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} =$

(d) $\lim_{x\to a}\dfrac{1}{f(x)} =$

(e) $\lim_{x\to a}\dfrac{f(x)}{h(x)} =$

(f) $\lim_{x\to a}\dfrac{g(x)}{f(x)} =$

(g) $\lim_{x\to a}\dfrac{f(x)}{g(x)} =$

(h) $\lim_{x\to a}\dfrac{2f(x)}{h(x)-f(x)} =$

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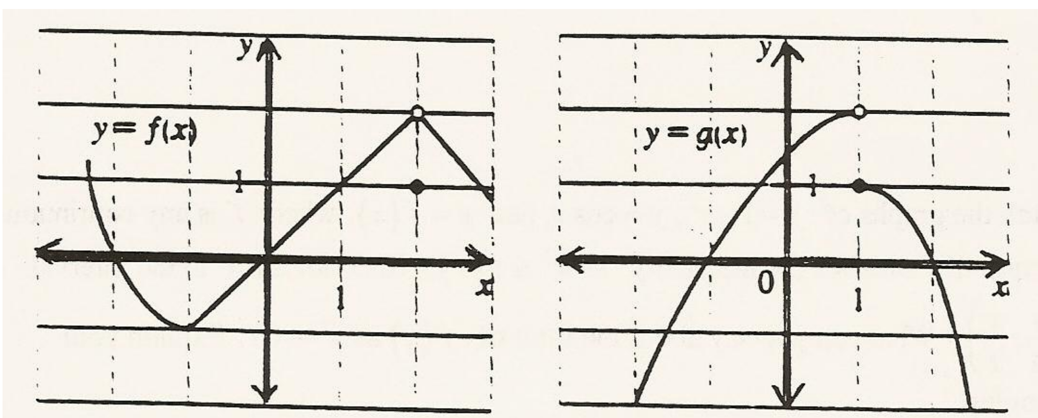
2. The graphs of $f$ and $g$ are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.




(a) $\lim_{x\to 2}\left[f(x)+g(x)\right] =$

(b) $\lim_{x\to 1}\left[f(x)+g(x)\right] =$

(c) $\lim_{x\to 0}\left[f(x)g(x)\right] =$

(d) $\lim_{x\to -1}\dfrac{f(x)}{g(x)} =$

(e) $\lim_{x\to 2} x^3 f(x) =$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} =$

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Find the following limits. Show all steps.

3. $\lim_{x\to 0}\dfrac{\sin 2x}{x} =$

**TURN--->>>**

4. $\lim_{x\to 0} \frac{\sin x}{2x^2 - x} =$

5. $\lim_{x\to 0} \frac{x+\sin x}{x} =$

6. $\lim_{x\to 0} \frac{\sin^2 x}{x} =$

7. $\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x} =$

8. $\lim_{x\to 0} \frac{x^2}{1-\cos x} =$

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9. Graph $y = x$, $y = -x$, and $y = x\cos\left(\frac{50\pi}{x}\right)$ on the same graph over the $x$-interval from $-1$ to 1, and use the Squeeze Theorem to find $\lim_{x\to 0} x\cos\left(\frac{50\pi}{x}\right)$.

10. Sketch the graphs of $y = 1 - x^2$, $y = \cos x$, and $y = f(x)$, where $f$ is any continuous function that satisfies the inequality $1 - x^2 \le f(x) \le \cos x$ for all $x$ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. What can you say about the limit of $f(x)$ as $x \to 0$? Explain your reasoning.

11. If $1 \le f(x) \le x^2 + 2x + 2$ for all $x$, find $\lim_{x\to -1} f(x)$.

12. If $3x \le f(x) \le x^3 + 2$, evaluate $\lim_{x\to 1} f(x)$.