Name KEY Date _____ Period

AP Calculus: Chain Rule Practice

Part I: Free Response. Find the first derivative for each, then simplify as much as possible

1. $f(x)=(2x+1)^3$, $f'(x)=3(2x+1)^2(2)=\boxed{6(2x+1)^2}$

2. $f(x)=\dfrac{1}{(x^2+4)^2}=(x^2+4)^{-2}$, $f'(x)=-2(x^2+4)^{-3}(2x)=\boxed{\dfrac{-4x}{(x^2+4)^3}}$

3. $h(x)=\cos(2t^2)+2\cos^2 t=\cos(2t^2)+2(\cos t)^2$, $h'(x)=-4t\sin(2t^2)+4\cos t(-\sin t)=\boxed{-4t\sin 2t^2-4\cos t\sin t}$

4. $h(u)=(3u^2+5)^3(3u-1)^2$, $h'(u)=3(3u^2+5)^2(6u)(3u-1)^2+(3u^2+5)^3(2)(3u-1)^1(3)=6(3u^2+5)^2(3u-1)\left[3u(3u-1)+(3u^2+5)\right]$
$=\boxed{6(3u^2+5)^2(3u-1)(12u^2-3u+5)}$

5. $f(y)=\left(\dfrac{y-7}{y+2}\right)^2$, $f'(y)=2\left(\dfrac{y-7}{y+2}\right)^1\cdot\left(\dfrac{(y+2)(1)-(y-7)(1)}{(y+2)^2}\right)=\dfrac{2(y-7)(y+2-y+7)}{(y+2)^3}=\dfrac{2(y-7)(9)}{(y+2)^3}=\boxed{\dfrac{18(y-7)}{(y+2)^3}}$

6. $f(r)=(r^2+1)^3(2r+5)^2$, $f'(r)=3(r^2+1)^2(2r)(2r+5)^2+(r^2+1)^3(2)(2r+5)^1(2)=2(r^2+1)^2(2r+5)\left[3r(2r+5)+2(r^2+1)\right]$
$=2(r^2+1)(2r+5)(6r^2+15r+2r^2+2)$
$=\boxed{2(r^2+1)(2r+5)(8r^2+15r+2)}$

7. $f(x)=4\cos(\sin 3x)$ $f'(x)=-4\sin(\sin(3x))\cdot\cos(3x)\cdot 3=\boxed{-12\cos(3x)\sin(\sin 3x)}$

8. $g(x)=\sqrt{1+4x^2}=(1+4x^2)^{1/2}$, $g'(x)=\frac{1}{2}(1+4x^2)^{-1/2}(8x)=\boxed{\dfrac{4x}{\sqrt{1+4x^2}}}$

9. $g(y)=\dfrac{1}{\sqrt{25-y^2}}=(25-y^2)^{-1/2}$, $g'(y)=-\frac{1}{2}(25-y^2)^{-3/2}(-2y)=\boxed{\dfrac{y}{(25-y^2)^{3/2}}}$

10. $f(x)=\dfrac{1}{(\sin 3x)^{\frac{1}{2}}}=(\sin(3x))^{-1/2}$, $f'(x)=-\frac{1}{2}(\sin(3x))^{-3/2}\cdot\cos(3x)\cdot 3=\boxed{\dfrac{-3\cos(3x)}{2\sqrt{\sin^3(3x)}}}$

11. $f(x)=\sqrt[3]{2x^3-5x^2+x}=(2x^3-5x^2+x)^{1/3}$, $f'(x)=\frac{1}{3}(2x^3-5x^2+x)^{-2/3}\cdot(6x^2-10x+1)=\dfrac{6x^2-10x+1}{3(2x^3-5x^2+x)^{2/3}}$

12. $h(y)=\cos\sqrt{y^2+1}=\cos((y^2+1)^{1/2})$, $h'(y)=-\sin\left(\sqrt{y^2+1}\right)\cdot\left(\frac{1}{2}(y^2+1)^{-1/2}\right)\cdot 2y=\boxed{\dfrac{-y\sin\sqrt{y^2+1}}{\sqrt{y^2+1}}}$

13. $g(x)=\sqrt{1+\cos^2 2x}=\left(1+(\cos(2x))^2\right)^{1/2}$; $g'(x)=\frac{1}{2}\left(1+\cos^2 2x\right)^{-1/2}\cdot\left(0+2(\cos(2x))^1\cdot(-\sin(2x)\cdot 2)\right)$
$=\boxed{\dfrac{-2\cos(2x)\sin(2x)}{\sqrt{1+\cos^2(2x)}}}$

14. $f(x)=\cos^2 x^2$
$=\left(\cos(x^2)\right)^2$
$f'(x)=2\left(\cos(x^2)\right)^1\cdot(-\sin(x^2))\cdot(2x)$
$=\boxed{-4x\cos(x^2)\sin(x^2)}$

MC. Answers: (1) D (2) E (3) A (4) E (5) B (6) B (7) A (8) A (9) C (10) B