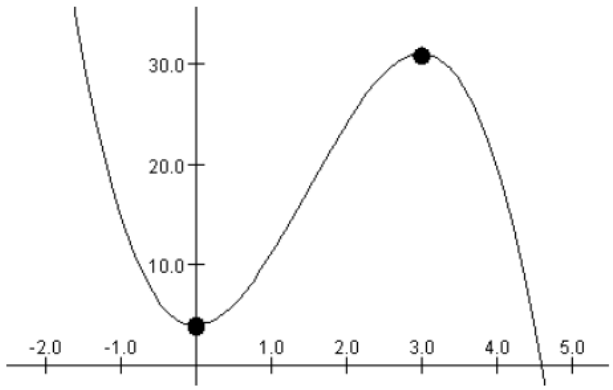
Notes: Applications of Derivatives
CURVE SKETCHING

Section 4.1: Extrema on an interval



The graph of $f(x) = -2x^3 + 9x^2 + 4$ is shown at left. Using the graph, write down the open x-intervals over which $f(x)$ is

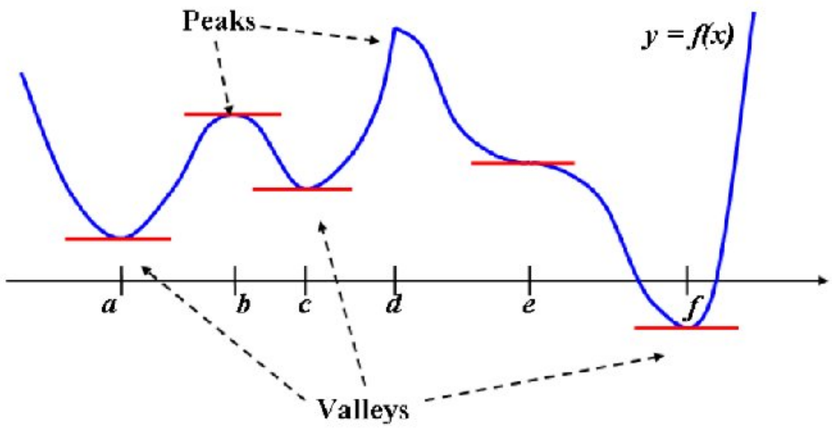
Increasing:

Decreasing:

Increasing and Decreasing Functions.

1. $f(x)$ is **increasing** on an interval if $f'(x) > 0$ on that interval.
2. $f(x)$ is **decreasing** on an interval if $f'(x) < 0$ on that interval.

Example 1. Use the above criteria to determine (verify) the intervals where $f(x) = -2x^3 + 9x^2 + 4$ is increasing and decreasing.



A "Peak" is a **Relative/Local Maximum** of *f*

A "Valley" is **a Relative/Local Minimum** of *f*

Notes:

1. Relative Max and Mins are *y*-values. They occur at *x*-values
2. Together, Relative Max and Relative Mins are called **Relative/Local Extrema**

Using the graph above, *f* has

Relative Maxima at $x =$

Relative Minima at $x =$

Critical Numbers/Values and Critical Points.
An *x*-value *c* in the domain of $f(x)$ is called a **critical number/value** if EITHER $f'(c)=0$ OR $f'(c)$ does not exist. The corresponding point $\left(c, f(c)\right)$ is called a critical point.

From the graph above, *f* has critical numbers at $x =$

Notes:

1. Relative extrema can only occur on OPEN intervals (not endpoints)*
2. Relative extrema can only occur at critical points.
3. Not all critical points correspond to Relative Extrema.

Example 1 (continued). Find the critical points of $f(x) = -2x^3 + 9x^2 + 4$

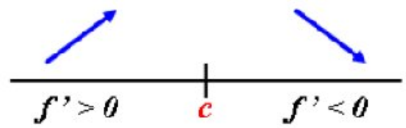
Example 2. Find the critical points of $f(x) = \left(x-1\right)^{2/3} + 2$

* Depends who you ask!

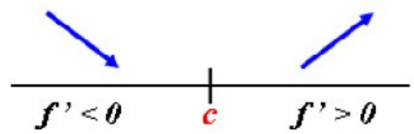
The First Derivative Test or Relative Extrema. (A process for determining which critical values are actually relative extrema of $f(x)$ based on first derivative information)

Steps:

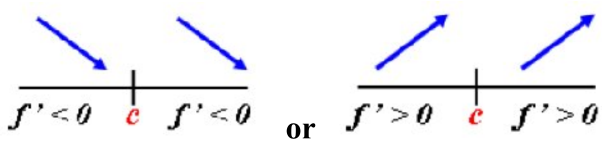
1. Differentiate $f(x)$ to find the critical values c of $f(x)$.
2. Set up a number line chart testing between all critical values (and any **discontinuities**).
3. Select convenient values from the created intervals, then plug into the FACTORED form of $f'(x)$ (if possible) to determine the SIGN of the derivative on that interval.
4. Draw your conclusion: at each critical value, based upon the following, $(c, f(c))$ is
   a. A **Relative Maximum** if $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$.



   b. A **Relative Minimum** if $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$.



   c. Not a Relative Extremum if $f'(x)$ has the same sign on both sides of $x = c$.



5. **(IMPORTANT) Write a concluding statement discussing the type of Relative Extrema each critical value might be based up the appropriate sign change (or not) of $f'(x)$ at $x = c$.**

<u>Example 2 (continued)</u>. Find the relative extrema of $f(x) = (x-1)^{2/3} + 2$, then sketch the graph.

For each of the following examples, find and justify the relative extrema of each function, if they exist. Sketch a graph of the function. Also, list the open intervals over which the function is increasing and/or decreasing.

Example 3. $f(x) = x^3 - 3x^2 + 3x - 1$

Example 4. $f(x) = \frac{5}{x^2 - 1}$

Example 5. Find the critical points of $f(x) = x + \sin 2x$ in the interval $[0, 2\pi]$. Determine what kind (if any) which kind of Relative Extremum each critical point is. Be sure to justify. Sketch the graph of $f(x)$ on the interval $[0, 2\pi]$.