Name KEY Date Period

Derivative Practice: All Rules

Find the indicated derivative in each case. You should try to simplify your answers if you can, including factoring when possible.

1. $f'(t)$ for $f(t)=\dfrac{t^2}{\sqrt{t+1}}$, $f' = \dfrac{\sqrt{t+1}(2t) - t^2\left(\frac{1}{2}(t+1)^{-1/2}\right)}{t+1} = \dfrac{2t\sqrt{t+1} - \frac{t^2}{2\sqrt{t+1}}}{t+1}\left(\dfrac{2\sqrt{t+1}}{2\sqrt{t+1}}\right)$

$$\boxed{f'(t) = \frac{3t^2+4t}{2(t+1)^{3/2}}}$$

2. $f'(x)$ for $f(x)=\dfrac{x^2+1}{x^3} = x^{-1} + x^{-3}$

$$\boxed{f'(x) = -\frac{1}{x^2} - \frac{3}{x^4}}$$ or $f'(x) = -\dfrac{x^4+3x^2}{x^6}$

$$\boxed{f'(x) = -\frac{x^2+3}{x^4}}$$

3. $\dfrac{dz}{dx}$ for $z=(x+1)^3(5-x)^4$ $z' = 3(x+1)^2(5-x)^4 + (x+1)^3(4)(5-x)^3(-1)$

$z' = (x+1)^2(5-x)^3[15-3x-4x-4]$

$$\boxed{z' = (x+1)^2(5-x)^3(11-7x)}$$

4. $f'(\theta)$ for $f(\theta)=\dfrac{1}{\tan(2\theta)} = \cot(2\theta)$

$f'(\theta) = -\csc^2(2\theta)\cdot 2$

$$\boxed{f'(\theta) = -2\csc^2(2\theta)}$$

5. $f''(x)$ for $f(x)=3x\cdot 2^{5x}$ $f' = 3\cdot 2^{5x} + 3x\cdot 2^{5x}\cdot \ln 2\cdot 5$

$f' = 3\cdot 2^{5x}(1+5\ln 2(x))$

$f'' = 3\cdot 2^{5x}\cdot \ln 2\cdot 5(1+5\ln 2(x)) + 3\cdot 2^{5x}(5\ln 2)$

$$\boxed{f'' = 15\ln 2\cdot 2^{5x}(2+5\ln 2(x))}$$

6. $f'(\beta)$ for $f(\beta)=\dfrac{\beta y + y^6}{1-\beta}$ * treat y as a constant

$f' = \dfrac{(1-\beta)(y) - (\beta y + y^6)(0)}{(1-\beta)^2} = \dfrac{(1-\beta)y}{(1-\beta)^2}$

$$\boxed{f' = \frac{y}{1-\beta}}$$

7. $\dfrac{dy}{dt}$ for $y=\ln(\ln(2t^3))$, $y' = \dfrac{1}{\ln(2t^3)}\cdot\dfrac{1}{2t^3}\cdot 6t^2$

$$\boxed{y' = \frac{3}{t\ln(2t^3)}}$$

8. $g'(x)$ for $g(x)=x\cdot e^{x^2}$

$g' = e^{x^2} + x\cdot e^{x^2}\cdot 2x$

$$\boxed{g' = e^{x^2}(1+2x^2)}$$

9. $x'(r)$ for $x(r)=3\sqrt[3]{r} - \sqrt{\dfrac{3}{r}} + \dfrac{1}{3r} = 3r^{1/3} - \sqrt{3}\,r^{-1/2} + \frac{1}{3}r^{-1}$

$x'(r) = r^{-2/3} + \frac{\sqrt{3}}{2}r^{-3/2} - \frac{1}{3}r^{-2}$

$$\boxed{x'(r) = \frac{1}{\sqrt[3]{r^2}} + \frac{\sqrt{3}}{2\sqrt{r^3}} - \frac{1}{3r^2}}$$

10. $h'(y)$ for $h(y)=\dfrac{\cos y}{1-\sin y}$ $h'(y) = \dfrac{(1-\sin y)(-\sin y) - \cos y(-\cos y)}{(1-\sin y)^2}$

$h'(y) = \dfrac{-\sin y + \sin^2 y + \cos^2 y}{(1-\sin y)^2} = \dfrac{1-\sin y}{(1-\sin y)^2}$

$$\boxed{h'(y) = \frac{1}{1-\sin y}}$$

11. $\dfrac{dz}{dx}$ for $z=10^{2\log x} = 10^{\log_{10} x^2} = x^2$

$$\boxed{\frac{dz}{dx} = 2x}$$

12. $f'(x)$ for $f(x)=\operatorname{arcsec}(4x^2+1)$

$f' = \dfrac{1}{|4x^2+1|\sqrt{(4x^2+1)^2-1}}\cdot(8x) = \dfrac{8x}{(4x^2+1)\sqrt{16x^4+8x^2}}$

$= \dfrac{8x}{(4x^2+1)(2x)\sqrt{4x^2+2}} = \boxed{\dfrac{4}{(4x^2+1)\sqrt{4x^2+2}}}$

13. $f'(t)$ for $f(t)=\arctan\left(\dfrac{2}{t}\right)$, $f'(t) = \dfrac{1}{1+\left(\frac{2}{t}\right)^2}\cdot\left(\dfrac{-2}{t^2}\right)$

$$\boxed{f'(t) = \frac{-2}{t^2+4}}$$

14. $g'(\theta)$ for $g(\theta)=\sqrt{3\theta+\tan^2(4\theta)} = \left(3\theta + (\tan(4\theta))^2\right)^{1/2}$

$g'(\theta) = \frac{1}{2}(3\theta+\tan^2(4\theta))^{-1/2}\cdot(3+2\tan(4\theta)\cdot\sec^2(4\theta)\cdot 4)$

$$\boxed{g'(\theta) = \frac{3+8\tan(4\theta)\sec^2(4\theta)}{2\sqrt{3\theta+\tan^2(4\theta)}}}$$

15. $f'(x)$ for $f(x) = x\cos\left(e^x\right)$

$f'(x) = \cos(e^x) - x\sin(e^x)\cdot e^x$

$\boxed{f'(x) = \cos(e^x) - xe^x\sin(e^x)}$

16. implicit $\frac{d^2y}{du^2}$ for $x^2 - y^2 = 25$

$\frac{d}{dx}[x^2-y^2] = \frac{d}{dx}[25]$

$2x - 2yy' = 0$

$y' = \frac{x}{y}$

$\frac{d}{dx}[y'] = \frac{d}{dx}\left[\frac{x}{y}\right]$

$y'' = \frac{y(1) - x(y')}{y^2} = \frac{y - x(\frac{x}{y})}{y^2}\left(\frac{y}{y}\right) = \boxed{\frac{y^2 - x^2}{y^3} = y''}$

17. log diff required $g'(z)$ for $g(z) = (\sin z)^{2z+1}$ $\ln g(z) = (2z+1)\ln\sin z$

$\frac{1}{g(z)}\cdot g'(z) = (2)\ln\sin z + (2z+1)\left(\frac{1}{\sin z}\right)(\cos z)$

$\boxed{g'(z) = (\ln\sin^2 z + (2z+1)\cot z)(\sin z)^{2z+1}}$

18. $f'(x)$ for $f(x) = \frac{x^2}{(2+x)^3}$, $f'(x) = \frac{(2+x)^3(2x) - x^2\cdot 3(2+x)^2}{((2+x)^3)^2}$

$f'(x) = \frac{x(2+x)^2[2(2+x)-3x]}{(2+x)^6}$, $f'(x) = \frac{x(2+x)^2(4-x)}{(2+x)^6}$

$\boxed{f'(x) = \frac{4x - x^2}{(x+2)^4}}$

19. $a'(t)$ for $a(t) = \ln\left(\frac{1-\cos t}{1+\cos t}\right)^4 = 4\ln(1-\cos t) - 4\ln(1+\cos t)$

$a'(t) = 4\left(\frac{1}{1-\cos t}\right)(\sin t) - 4\left(\frac{1}{1+\cos t}\right)(-\sin t) = \frac{4\sin t}{1-\cos t} + \frac{4\sin t}{1+\cos t}$

$a'(t) = \frac{4\sin t + 4\sin t\cos t + 4\sin t - 4\sin t\cos t}{1-\cos^2 t} = \frac{8\sin t}{\sin^2 t}$

$\boxed{a'(t) = 8\csc t}$

20. $\left(g^{-1}\right)'(2)$ if $g(x) = 2x^3 - x^2 + 1$ $g(1) = 2 - 1 + 1 = 2$

$g^{-1}: (2, 1)$ $g'(x) = 6x^2 - 2x$

$g: (1, 2)$ $g'(1) = 6 - 2 = 4$

so $(g^{-1})'(2) = \frac{1}{g'(1)} = \boxed{\frac{1}{4}}$

21. $\frac{dy}{dx}$ for $x^2 + 2xy + \sin\left(y^2\right) = 3$

$\frac{d}{dx}[x^2 + 2xy + \sin(y^2)] = \frac{d}{dx}[3]$

$2x + 2y + 2xy' + 2yy'\cos(y^2) = 0$

$y' = \frac{-2x - 2y}{2x + 2y\cos(y^2)}$ $\boxed{y' = -\frac{x+y}{x + y\cos(y^2)}}$

22. log diff $f'(m)$ for $f(m) = (\sec m)^{\sqrt{m}}$

$\ln f(m) = (m^{1/2})\ln(\sec m)$

$\frac{f'(m)}{f(m)} = \frac{1}{2\sqrt{m}}\ln(\sec m) + \sqrt{m}\left(\frac{\sec m\tan m}{\sec m}\right)$

$\boxed{f'(m) = \left(\frac{\ln\sec m}{2\sqrt{m}} + \sqrt{m}\tan m\right)(\sec m)^{\sqrt{m}}}$

23. $g'(\theta)$ for $g(\theta) = \sqrt[3]{\tan(5\theta)} = (\tan(5\theta))^{1/3}$

$g'(\theta) = \frac{1}{3}(\tan(5\theta))^{-2/3}\cdot\sec^2(5\theta)\cdot 5$

$\boxed{g'(\theta) = \frac{5\sec^2(5\theta)}{3\sqrt[3]{\tan^2(5\theta)}}}$

24. $\frac{dy}{du}$ for $y = (\cot 1 + \cot u)^{\pi}$

$y' = \pi(\cot 1 + \cot u)^{\pi - 1}\cdot(-\csc^2 u)$

$\boxed{y' = -\pi\csc^2 u(\cot 1 + \cot u)^{(\pi - 1)}}$

25. $\frac{dy}{dt}$ for $y = \frac{x^3\sin^3 x}{(x+1)^2\sqrt[3]{\left(3x^3+x\right)^2}}$ log diff

$\ln y = 3\ln x + 3\ln(\sin x) - 2\ln(x+1) - \frac{2}{3}\ln(3x^3 + x)$

$\boxed{y' = \left(\frac{3}{x} + 3\cot x - \frac{2}{x+1} - \frac{18x^2+2}{9x^3+3x}\right)\frac{x^3\sin^3 x}{(x+1)^2\sqrt[3]{(3x^3+x)^2}}}$

26. $\left(q^{-1}\right)'(1)$ for $q(t) = 3t + \arccos t$ $q(0) = 0 + \arccos 0$ $q(0) = 1$

$q'(t) = 3 - \frac{1}{\sqrt{1-t^2}}$, $q'(0) = 2$

so $(q^{-1})'(1) = \frac{1}{q'(0)} = \boxed{\frac{1}{2}}$