

AB Calculus Practice Test: Particle Motion through Implicit Differentiation

Part I: Multiple Choice

1. (Calculator Active)

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

- (A) $y = 8x - 5$
- (B) $y = x + 7$
- (C) $y = x + 0.763$
- (D) $y = x - 0.122$
- (E) $y = x - 2.146$

2.

If $\frac{dy}{dx} = \sqrt{1-y^2}$, then $\frac{d^2y}{dx^2} =$

- (A) $-2y$
- (B) $-y$
- (C) $\frac{-y}{\sqrt{1-y^2}}$
- (D) y
- (E) $\frac{1}{2}$

3.

The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

- (A) $-\frac{3}{2}$
- (B) $-\frac{3}{4}$
- (C) 0
- (D) $\frac{3}{4}$
- (E) $\frac{3}{2}$

4.

If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

- (A) $\sqrt{3}$
- (B) $2\sqrt{3}$
- (C) 4
- (D) $4\sqrt{3}$
- (E) 8

5.

If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

(A) $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

(B) $f''(g(x))g'(x) + f'(g(x))g''(x)$

(C) $f''(g(x))[g'(x)]^2$

(D) $f''(g(x))g''(x)$

(E) $f''(g(x))$

6.

Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

- I. f has at least 2 zeros.
- II. The graph of f has at least one horizontal tangent.
- III. For some c , $2 < c < 5$, $f(c) = 3$.

- (A) None
- (B) I only
- (C) I and II only
- (D) I and III only
- (E) I, II, and III

7.

A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8.

What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

- (A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

9.

If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

- (A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

10.

Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

- (A) $f(0) = 0$
(B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6
(C) $-1 \leq f(x) \leq 3$ for all x between -3 and 6
(D) $f(c) = 1$ for at least one c between -3 and 6
(E) $f(c) = 0$ for at least one c between -1 and 3

11. (Calculator Active)

The position of an object attached to a spring is given by $y(t) = \frac{1}{6} \cos(5t) - \frac{1}{4} \sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

- (A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

12.

The line normal to the curve $y = \sqrt{16-x}$ at the point $(0,4)$ has slope

- (A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) -8

13.

Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

- (A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

14.

If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

- (A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

15.

At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

- (A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

16.

If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

- (A) 3 (B) 1 (C) -1 (D) -3 (E) -5

17.

The value of the derivative of $y = \frac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$ at $x = 0$ is

- (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

18.

An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

19.

$$\frac{d}{dx} \cos^2(x^3) =$$

(A) $6x^2 \sin(x^3) \cos(x^3)$

(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3) \cos(x^3)$

(E) $-2 \sin(x^3) \cos(x^3)$

20.

If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{2\sqrt{2x-3}}$

21.

If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a+c =$

- (A) -5 (B) -1 (C) 0 (D) 1 (E) 5

22. (Calculator active)

A particle moves along a line so that at time t , where $0 \leq t \leq \pi$, its position is given by

$s(t) = -4 \cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

- (A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

23.

If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

- (A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

24.

If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

25.

If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

- (A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

26.

An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point (1,5) is

- (A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$
(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

27.

If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

- (A) -6 (B) -3 (C) 3 (D) 6 (E) 8

28.

If $x + 7y = 29$ is an equation of the line normal to the graph of f at the point (1,4), then $f'(1) =$

- (A) 7 (B) $\frac{1}{7}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{29}$ (E) -7

29.

If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y , $\frac{dy}{dx} =$

(A) $-\frac{x^2 + y}{x + 2y^2}$

(B) $-\frac{x^2 + y}{x + y^2}$

(C) $-\frac{x^2 + y}{x + 2y}$

(D) $-\frac{x^2 + y}{2y^2}$

(E) $\frac{-x^2}{1 + 2y^2}$

30.

$\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$ is

- (A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

31.

If $y^2 - 2xy = 16$, then $\frac{dy}{dx} =$

- (A) $\frac{x}{y-x}$ (B) $\frac{y}{x-y}$ (C) $\frac{y}{y-x}$ (D) $\frac{y}{2y-x}$ (E) $\frac{2y}{x-y}$

32.

If u , v , and w are nonzero differentiable functions, then the derivative of $\frac{uv}{w}$ is

- (A) $\frac{uv' + u'v}{w'}$ (B) $\frac{u'v'w - uvw'}{w^2}$ (C) $\frac{uvw' - uv'w - u'vw}{w^2}$
(D) $\frac{u'vw + uv'w + uvw'}{w^2}$ (E) $\frac{uv'w + u'vw - uvw'}{w^2}$

33.

The $\lim_{h \rightarrow 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

- (A) 0 (B) $3 \sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3 \cot(3x)$ (E) nonexistent

34.

If $y = 2 \cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

- (A) $-8 \cos\left(\frac{x}{2}\right)$ (B) $-2 \cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2} \cos\left(\frac{x}{2}\right)$

AB Free Response:

1. (AB 2000)

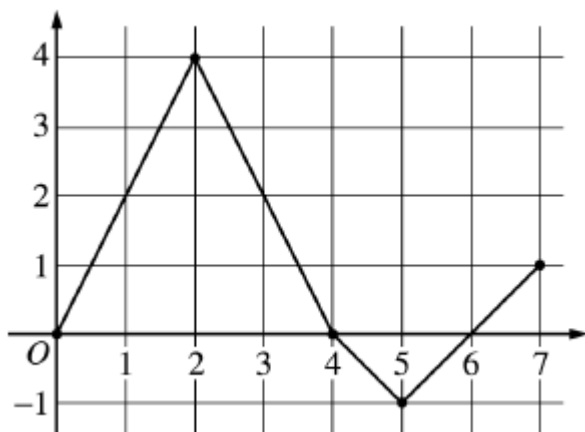
Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x -coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x -coordinate of each point on the curve where the tangent line is vertical.

1. A three-toed sloth is hanging vertically from its tail on a horizontal tree branch, high in the sky. The graph below describes the horizontal movement (in inches) of the sloth along the branch as it moves from left to right, with zero corresponding to the middle of the branch. Time is measured in hours along the x -axis.



- What is the displacement of the sloth during the first three hours? Is he to the left or right of the middle?
 - What is the sloth's average velocity during the first three hours?
 - What is the sloth's speed at hour 3? In which direction is he moving?
 - At what times does the sloth change directions?
 - What is the velocity of the sloth at hour 5?
 - What is the acceleration of the sloth between hours 5 and 7?
 - At what time is the sloth the furthest from the center of the branch?
 - During what times is the sloth moving to the right?
 - How many total inches did the sloth move during the 7 hour period?
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