Name____ Date____________________ Per________

Practice TEST II: AP Calculus: Test—Integration concepts (minus inverse trig and u-sub)

1. If $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, then $f(0)$ equals

   (A) 2 (B) 3 C) 4 (D) –1

2. The volume V of a balloon is changing with respect to time t at a rate given by $\frac{dV}{dt} = 3t^{1/2} + \frac{1}{4}t \;\; ft^3/\text{sec}$. If, at $t = 4$, the volume is $20\,ft^3$, then V equals

   (A) $\frac{3}{2}t^{-1/2} + \frac{t}{4} + 19$ (B) $2t^{3/2} + \frac{1}{8}t^2 + 2$ (C) $\frac{9}{2}t^{3/2} + \frac{1}{2}t^2 - 24$ (D) $\frac{9}{4}t^{3/2} + \frac{1}{2}t^2 - 6$ (E) None of these

3. The most general antiderivative of $f(x) = (\cos 3x)(\tan 3x)$ is

   (A) $\frac{1}{3}\tan 3x + C$ (B) $\frac{1}{3}\cot 3x + C$ (C) $-\frac{1}{3}\cos 3x + C$ (D) $\frac{1}{3}\cos 3x + C$ (E) None of these

4. Evaluate the given integral by interpreting it as an area: $\int_0^3 (9 - x^2)^{1/2}\,dx$

   (A) 6π (B) 9π (C) $\frac{9\pi}{4}$ (D) $\frac{9\pi}{2}$

5. If $f(x) = (3x^3 + 2)$, then find a number c between –1 and 2 such that $\int_{-1}^{2} f(x)dx = 3f(c)$

   (A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) $\left(\frac{5}{4}\right)^{1/3}$ (D) $\left(\frac{4}{5}\right)^{1/3}$

9. $\int_1^2 (t^3 - 1)^{1/2} t^2\,dt$ equals

   (A) $7^{3/2}$ (B) $\frac{2}{9}(7)^{3/2}$ (C) $\frac{1}{2}(7^{-1/2})$ (D) $-\frac{2}{9}(7)^{3/2}$ (E) None of these

6. The average value of $f(x) = x^2 + 3x - 1$ on $[-1,2]$ equals

   (A) 3 (B) 2 (C) 1.5 (D) 1 (E) None of these

7. If f is continuous for all x, which of the following integrals necessarily have the same value?

   I. $\int_a^b f(x)dx$

   II. $\int_0^{b-a} f(x+a)dx$

   III. $\int_{a+c}^{b+c} f(x+c)dx$

   (A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) None

8. The table below gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$.
   a. What is this estimate?
   b. What is the error between using 5 rectangles vs. 5 **<u>trapezoids</u>**?
   c. What was the average flow rate during the 60 second time period (use the trapezoidal area)

Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

9. Find all possible values of k for which $\int_{-6}^{k} \left(x^3 - 3x\right)dx = 0$

10. $\int_0^5 |3x - 9|dx =$

T or F (if false, explain why or give a counterexample)

11. If $f(t) = \dfrac{\text{calories consumed}}{\text{week}}$ and $dt = \text{week}$, then the definite integral would give total calories consumed over a particular time period.

Evaluate the following Indefinite Integrals. Don't forget your +C

12. $\int \dfrac{2t^2 + 3t - 1}{\sqrt{t}} dt$

13. $\int 4 \cdot 3^x - \dfrac{4}{5 \cdot 6^{-x}} + \dfrac{6}{1 - \cos^2 x} dx$

14. $\int 6m^{-1} - \tan m \sec m + \dfrac{1}{7m} dm$

15. $\int \pi \sqrt[3]{x} \left(2x - 3\right)^2 dx$

32. If $f(x)$ is an even function and $\int_{-17}^{0} f(x)dx = 32.5$, find $2\int_{-17}^{17} f(x)dx$.

33. I collected some data yesterday and tried to use it to approximate a function $y = f(x)$.

x	**0**	**1/2**	**1**	**3/2**	**2**	**5/2**	**3**
y	**3**	**4**	**1**	**5**	**2**	**3**	**4**

Use my data to approximate $\int_0^3 f(x)dx$ using the following methods:

a. Left end-point Riemann Sums ($n = 6$)
b. Right end-point Riemann Sums ($n = 6$)
c. Midpoint Riemann Sums ($n = 3$)
d. Trapezoidal Rule ($n = 6$)
e. Approximate $f'(1)$ from the table of values.