Name_____ Date_____ Period_____

Practice TEST: All integration techniques and Differential Equations

NO CALCULATOR PERMITTED ON MULTIPLE CHOICE!!!!!!!

_____ 1.

The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

_____ 2.

If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln\left|\sec x\right| + C$

(D) $\ln\left|\cos x\right| + C$ (E) $\sec x \tan x + C$

_____ 3.

Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

_____ 4.

The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

_____ 5.

If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

_____ 6.

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

_____ 7.

If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

_____ 8. (CALCULATOR PERMITTED ON THIS ONE)

During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

_____ 9.

If f is continuous on the interval $[a,b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x)dx =$

(A) $\frac{f(c)}{b-a}$ (B) $\frac{f(b)-f(a)}{b-a}$ (C) $f(b)-f(a)$ (D) $f'(c)(b-a)$ (E) $f(c)(b-a)$

_____ 10.

If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

(A) 6 (B) 3 (C) 0 (D) -1 (E) -6

_____ 11.

If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

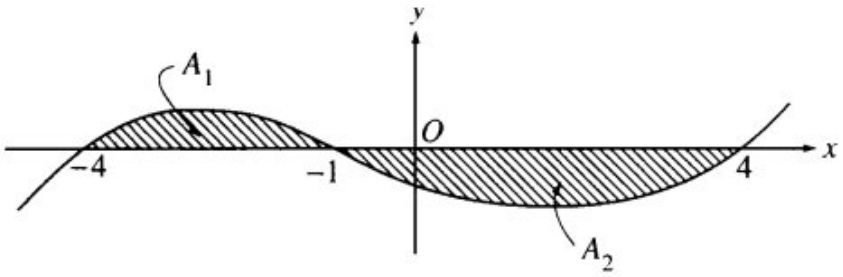
I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b \big(f(x) + g(x)\big)dx = 4$

III. $\int_a^b \big(f(x)g(x)\big)dx = -5$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

_____ 12.



The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) A_1 (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

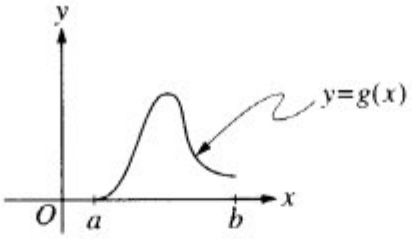
_____ 13.

x	2	5	7	8
$f(x)$	10	30	40	20

The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

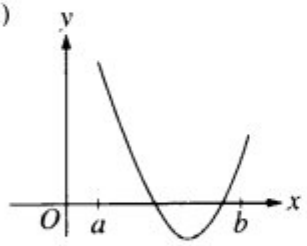
(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

_____ 14.

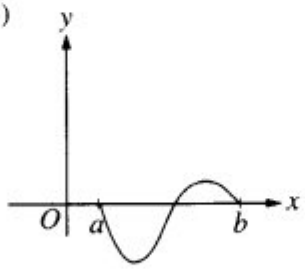


Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

(A)



(B)



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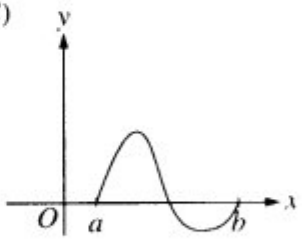
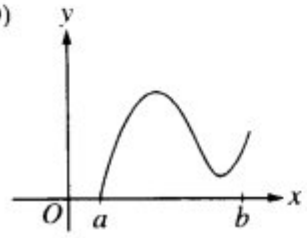
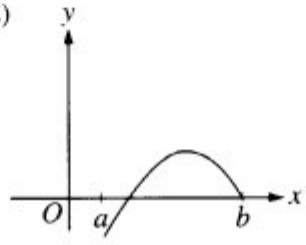


(D)



(E)



_____ 15. CALCULATOR PERMITTED FOR THIS ONE TOO

A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

_____ 16.

$$\int \frac{x^2}{e^{x^3}}\,dx =$$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

_____ 17.

$\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

_____ 18.

$\int (x^3-3x)\,dx =$

(A) $3x^2-3+C$ (B) $4x^4-6x^2+C$ (C) $\frac{x^4}{3}-3x^2+C$

(D) $\frac{x^4}{4}-3x+C$ (E) $\frac{x^4}{4}-\frac{3x^2}{2}+C$

_____ 19.

$\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

_____ 20.

$\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

_____ 21.

$\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

_____ 22.

$\int_0^1 xe^{-x}\,dx =$

(A) $1-2e$ (B) -1 (C) $1-2e^{-1}$ (D) 1 (E) $2e-1$

_____ 23.

$\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$ (E) $5\ln\left(1+x^2\right)+C$

_____ 24.

If $\frac{dy}{dx}=\cos(2x)$, then $y=$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

_____ 25.

$\int_1^2 \frac{x^2-1}{x+1}\,dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

_____ 26.

If $\int_{-2}^{2}\left(x^7+k\right)dx=16$, then $k=$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

_____ 27.

$\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

_____ 28.

If $\int_0^k\left(2kx-x^2\right)dx=18$, then $k=$

(A) –9 (B) –3 (C) 3 (D) 9 (E) 18

_____ 29.

$\int \tan(2x)\,dx =$

(A) $-2\ln\left|\cos(2x)\right|+C$ (B) $-\frac{1}{2}\ln\left|\cos(2x)\right|+C$ (C) $\frac{1}{2}\ln\left|\cos(2x)\right|+C$

(D) $2\ln\left|\cos(2x)\right|+C$ (E) $\frac{1}{2}\sec(2x)\tan(2x)+C$

_____ 30.

$\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

_____ 31.

$\int_{-1}^{1} \frac{3}{x^2}\,dx$ is

(A) –6 (B) –3 (C) 0 (D) 6 (E) nonexistent

_____ 32.

$\int \sec^2 x\,dx =$

(A) $\tan x+C$ (B) $\csc^2 x+C$ (C) $\cos^2 x+C$

(D) $\frac{\sec^3 x}{3}+C$ (E) $2\sec^2 x\tan x+C$

_____ 33.

$\int_0^1 (3x-2)^2\,dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

_____ 34.

$$\int \frac{x\,dx}{\sqrt{3x^2+5}} =$$

(A) $\frac{1}{9}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (B) $\frac{1}{4}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (C) $\frac{1}{12}\left(3x^2+5\right)^{\frac{1}{2}}+C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{\frac{1}{2}}+C$ (E) $\frac{3}{2}\left(3x^2+5\right)^{\frac{1}{2}}+C$

_____ 35.

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

_____ 36.

$$\int_2^3 \frac{x}{x^2+1}\,dx =$$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

_____ 37.

For $x>0$, $\int \left(\frac{1}{x}\int_1^x \frac{du}{u}\right) dx =$

(A) $\frac{1}{x^3}+C$ (B) $\frac{8}{x^4}-\frac{2}{x^2}+C$ (C) $\ln\left(\ln x\right)+C$

(D) $\frac{\ln\left(x^2\right)}{2}+C$ (E) $\frac{\left(\ln x\right)^2}{2}+C$

_____ 38.

If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^3 f(x)\,dx = 7$, then $\int_1^3 f(x)\,dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11

_____ 39.

If $F(x)=\int_1^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$

(D) $\sqrt{1+x^3}$ (E) $\int_1^{x^2}\frac{3t^2}{2\sqrt{1+t^3}}dt$

_____ 40. (hint: sketch the function)

$\int_0^2\sqrt{4-x^2}\,dx=$

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

_____ 41.

Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

_____ 42.

$\int (x^2+1)^2dx=$

(A) $\frac{(x^2+1)^3}{3}+C$

(B) $\frac{(x^2+1)^3}{6x}+C$

(C) $\left(\frac{x^3}{3}+x\right)^2+C$

(D) $\frac{2x(x^2+1)^3}{3}+C$

(E) $\frac{x^5}{5}+\frac{2x^3}{3}+x+C$

_____ 43.

An antiderivative for $\dfrac{1}{x^2-2x+2}$ is

(A) $-(x^2-2x+2)^{-2}$

(B) $\ln(x^2-2x+2)$

(C) $\ln\left|\dfrac{x-2}{x+1}\right|$

(D) $\operatorname{arcsec}(x-1)$

(E) $\arctan(x-1)$

_____ 44.

$\displaystyle\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{1}{2}\ln 2$ (E) $-\ln 2$

_____ 45.

If $\dfrac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\dfrac{2}{3}$ (B) $-\dfrac{1}{3}$ (C) 0 (D) $\dfrac{1}{3}$ (E) $\dfrac{2}{3}$

_____ 46.

$\dfrac{d}{dx}\displaystyle\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

_____ 47.

$\displaystyle\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\dfrac{1}{4}(e-1)$ (B) $\dfrac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

_____ 48.

If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

_____ 49.

Which of the following is equal to $\int_0^{\pi} \sin x\, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ (B) $\int_0^{\pi} \cos x\, dx$ (C) $\int_{-\pi}^{0} \sin x\, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx$ (E) $\int_{\pi}^{2\pi} \sin x\, dx$

_____ 50.

If $\frac{dy}{dx} = \frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1, 4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

_____ 51.

$\frac{1}{2}\int e^{\frac{t}{2}}\, dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

_____ 52.

$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

_____ 53.

$\int_0^1 \sqrt{x}(x+1)\, dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

_____ 54

Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

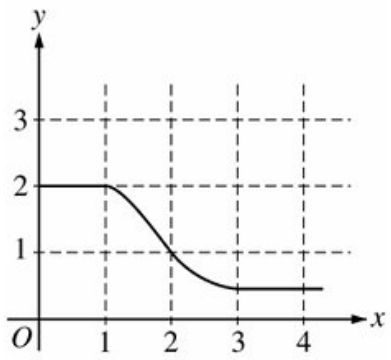
(B) II only

(C) III only

(D) I and III only

(E) II and III only

_____ 55.



The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

_____ 56.

$\int_0^x \sin t\,dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

_____ 57.

The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

_____ 58.

$\displaystyle\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e - \dfrac{1}{e}$ (B) $e^2 - e$ (C) $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ (D) $e^2 - 2$ (E) $\dfrac{e^2}{2} - \dfrac{3}{2}$

_____ 59.

If $F(x) = \displaystyle\int_0^x \sqrt{t^3+1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

_____ 60.

What are all values of k for which $\displaystyle\int_{-3}^{k} x^2 dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

_____ 61.

If $\dfrac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\dfrac{1}{2}ky^2 + \dfrac{1}{2}$

_____ 62.

What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

_____ 63.

If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

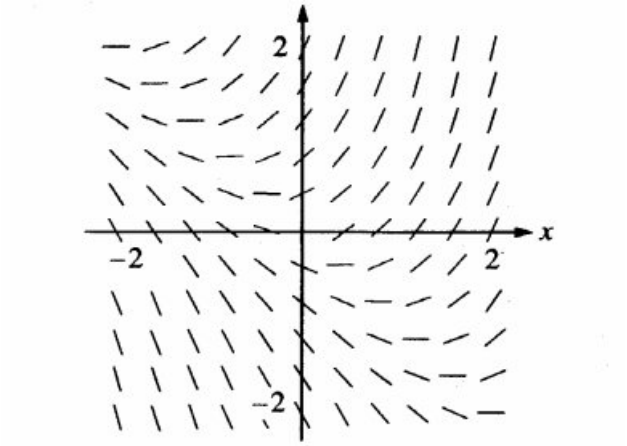
(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

_____ 64.

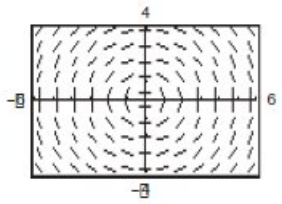


. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

_____ 65.

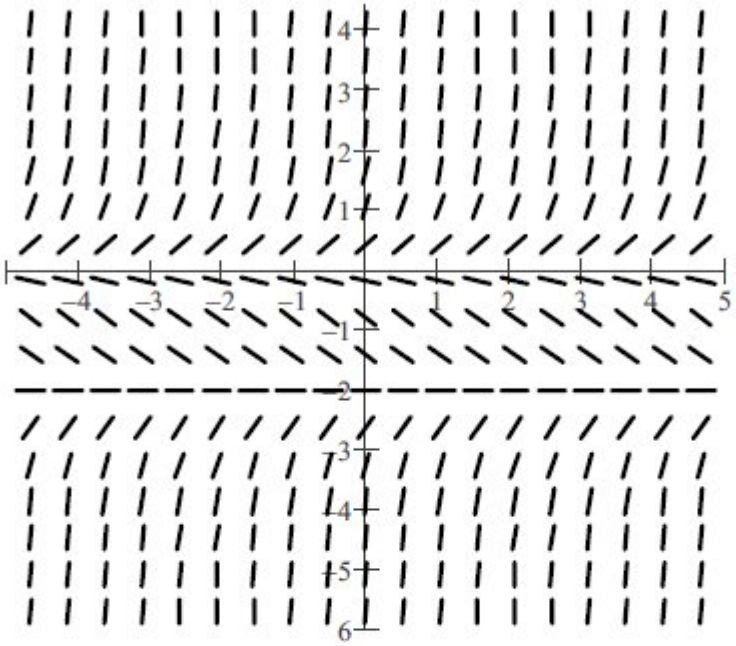
The slope field for a differential equation is shown in the figure. Determine the general solution of this equation.



(a) $y = Cx^2$ (b) $x = Cy^2$ (c) $x^2 - y^2 = C^2$

(d) $y^2 - x^2 = C^2$ (e) $x^2 + y^2 = C^2$

_____ 66.



Which statement is true about the solutions $y(x)$, of a differential equation whose slope field is shown above?

I. If $y(0) > 0$ then $\lim_{x\to\infty} y(x) \approx 0$.

II. If $-2 < y(0) < 0$ then $\lim_{x\to\infty} y(x) \approx -2$.

III. If $y(0) < -2$ then $\lim_{x\to\infty} y(x) \approx -2$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

_____ 67.

The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y + y^2}{4x + 2y}$ will have vertical segments when

(A) $y = 2x$, only

(B) $y = -2x$, only

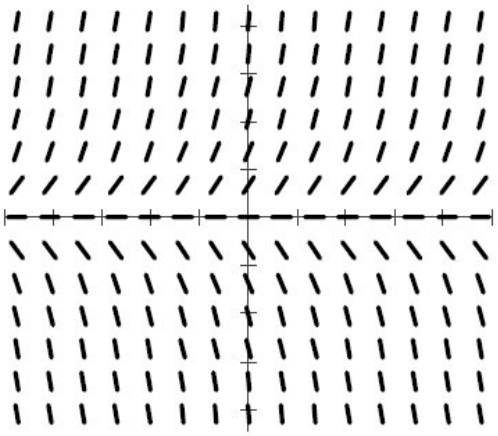
(C) $y = -x^2$, only

(D) $y = 0$, only

(E) $y = 0$ or $y = -x^2$

_____ 68.

The slope field for a differential equation is shown at the right. Which statement is true for solutions of the differential equation?



I. For $x < 0$ all solutions are decreasing.

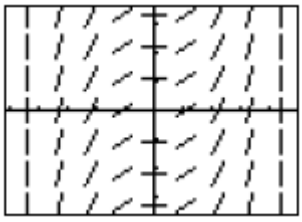
II. All solutions level off near the x-axis.

III. For $y > 0$ all solutions are increasing.

(A) I only (B) II only (C) III only

(D) II and III only (E) I, II, and III

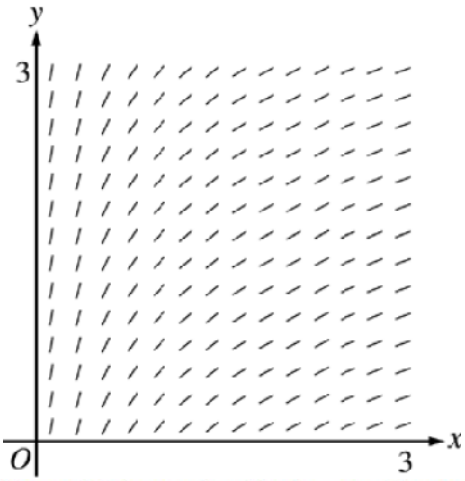
_____ 69.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

_____ 70.



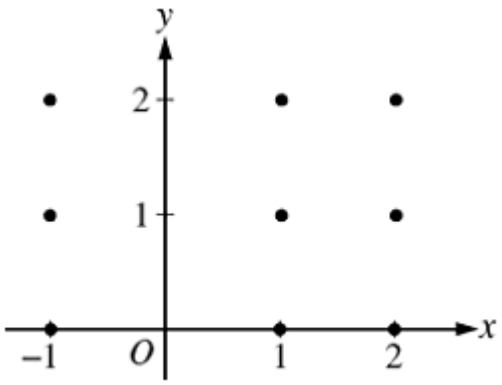
The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

Part II: Free Response:

1. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



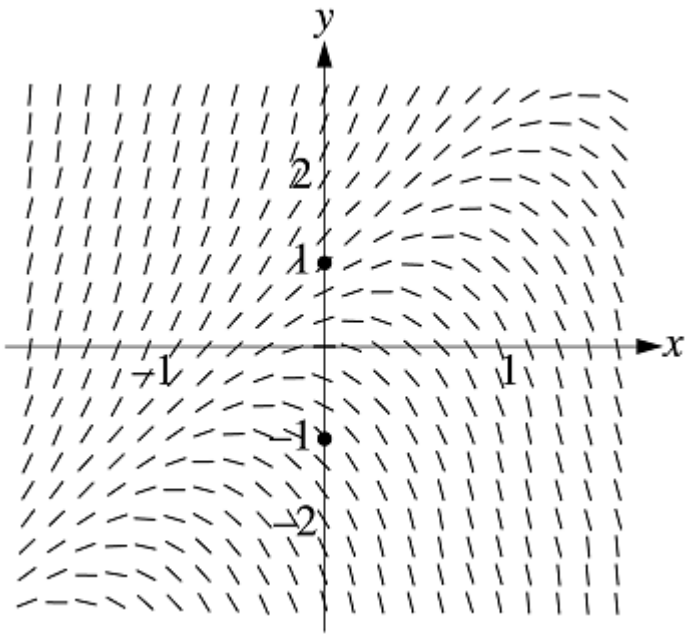
(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

2.

Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that passes through the point $(0,-1)$.



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Find the equation of the tangent line of f at (0.1), and use it to approximate $f(0.2)$. Is your approximation an over or under approximation? Justify your answer.

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0,0)$? If so, is the point a local maximum or a local minimum? Justify your answer.