AP Calculus AB
Review 15, No Calculator

Complete all the following on notebook paper.

_____ 1.
If $f(x_1)+f(x_2)=f(x_1+x_2)$ for all real numbers x_1 and x_2, which of the following could define f?

(A) $f(x)=x+1$ (B) $f(x)=2x$ (C) $f(x)=\frac{1}{x}$ (D) $f(x)=e^x$ (E) $f(x)=x^2$

_____ 2.
If $y=\arctan(\cos x)$, then $\frac{dy}{dx}=$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-\left(\operatorname{arcsec}(\cos x)\right)^2 \sin x$ (C) $\left(\operatorname{arcsec}(\cos x)\right)^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

_____ 3.
If the domain of the function f given by $f(x)=\frac{1}{1-x^2}$ is $\{x:|x|>1\}$, what is the range of f?

(A) $\{x:-\infty<x<-1\}$ (B) $\{x:-\infty<x<0\}$ (C) $\{x:-\infty<x<1\}$

(D) $\{x:-1<x<\infty\}$ (E) $\{x:0<x<\infty\}$

_____ 4.
$\int_1^2 \frac{x^2-1}{x+1}\,dx=$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

_____ 5.
If $\int_{-2}^{2}\left(x^7+k\right)dx=16$, then $k=$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

_____ 6.

$\frac{d}{dx}\left(\frac{1}{x^3}-\frac{1}{x}+x^2\right)$ at $x=-1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

_____ 7.

If $f(x)=e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0}\frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0}\frac{e^{x+h}-e^e}{h}$ (C) $\lim_{h\to 0}\frac{e^{e+h}-e}{h}$

(D) $\lim_{h\to 0}\frac{e^{x+h}-1}{h}$ (E) $\lim_{h\to 0}\frac{e^{e+h}-e^e}{h}$

_____ 8.

The graph of $y^2=x^2+9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

_____ 9.

$\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

_____ 10.

If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0\le t\le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

Free Response (No calculator)

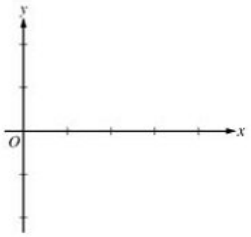
11. 2005-AB4

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.
(Note: Use the axes provided in the pink test booklet.)



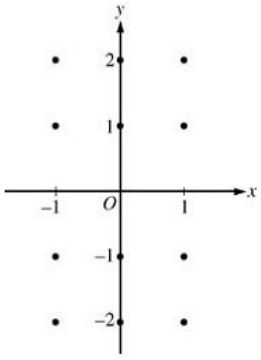
(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

12. 2005-AB6

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.