

AP Calculus AB

Review 16, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.

Which of the following functions are continuous for all real numbers x ?

I. $y = x^{\frac{2}{3}}$

II. $y = e^x$

III. $y = \tan x$

- (A) None (B) I only (C) II only (D) I and II (E) I and III

_____ 2.

$$\int \tan(2x) dx =$$

- (A) $-2 \ln |\cos(2x)| + C$ (B) $-\frac{1}{2} \ln |\cos(2x)| + C$ (C) $\frac{1}{2} \ln |\cos(2x)| + C$
(D) $2 \ln |\cos(2x)| + C$ (E) $\frac{1}{2} \sec(2x) \tan(2x) + C$

_____ 3.

The volume of a cone of radius r and height h is given by $V = \frac{1}{3} \pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

- (A) $\frac{1}{2} \pi$ (B) 10π (C) 24π (D) 54π (E) 108π

_____ 4.

$$\int_0^{\frac{\pi}{3}} \sin(3x) dx =$$

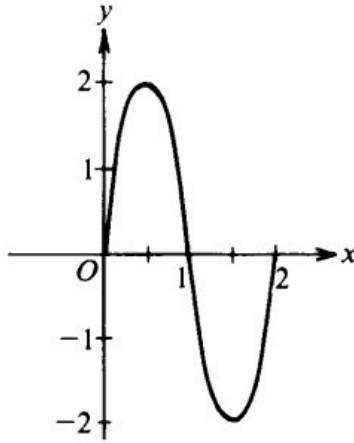
- (A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

_____ 5.

The area of the region in the first quadrant that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$

_____ 6.



The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

- (A) $y = 2 \sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2 \sin(2x)$
(D) $y = 2 \sin(\pi x)$ (E) $y = \sin(2x)$

_____ 7.

If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7 , then which of the following must be true?

- I. The maximum value of $f(|x|)$ is 5.
- II. The maximum value of $|f(x)|$ is 7.
- III. The minimum value of $f(|x|)$ is 0.

- (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 8.

$\lim_{x \rightarrow 0} (x \csc x)$ is

- (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

_____ 9.

Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x , which of the following must be true?

- I. $f'(x) \leq g'(x)$ for all real x
- II. $f''(x) \leq g''(x)$ for all real x
- III. $\int_0^1 f(x) dx \leq \int_0^1 g(x) dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

_____ 10.

If $f(x) = \frac{\ln x}{x}$, for all $x > 0$, which of the following is true?

- (A) f is increasing for all x greater than 0.
- (B) f is increasing for all x greater than 1.
- (C) f is decreasing for all x between 0 and 1.
- (D) f is decreasing for all x between 1 and e .
- (E) f is decreasing for all x greater than e .

Free Response (Calculator Permitted)

11. 2004-AB2B

For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

- Show that the number of mosquitoes is increasing at time $t = 6$.
- At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
- According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.
- To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

12. 1970-AB4

A right circular cone and a hemisphere have the same base, and the cone is inscribed in the hemisphere. The figure is expanding in such a way that the combined surface area of the hemisphere and its base is increasing at a constant rate of 18 square inches per second. At what rate is the volume of the cone changing at the instant when the radius of the common base is 4 inches? Show your work.