AP Cal ABC
§2.1
p. 66 (2-8 even, 14, 17, 18-30 even, 34-44 even, 48, 50, 52, 58, 65, 66)

Korpi
period 11teen

(2) $y = 16t^2$ feet falls in t sec.

Avg speed $0 \le t \le 4$ $= \dfrac{y(4) - y(0)}{4 - 0}$

$= \dfrac{16^2 - 0}{4}$

$= \dfrac{256}{4}$

$= \boxed{64 \text{ ft/sec}}$

(4) speed @ $t = 4$

$\dfrac{\Delta y}{\Delta t} = \dfrac{(16(4+h)^2) - (16 \cdot 4^2)}{h}$

$= \dfrac{16(16 + 8h + h^2) - 256}{h}$

$= \dfrac{\cancel{256} + 128h + 16h^2 - \cancel{256}}{h}$

$= \dfrac{\cancel{h}(128 + 16h)}{\cancel{h}}$

$= 128 + 16h$

$\boxed{\text{As } h \to 0, \text{ speed approaches } 128 \text{ ft/sec}}$

(6) $\lim\limits_{x \to c} \dfrac{x^4 - x^3 + 1}{x^2 + 9}$

$= \boxed{\dfrac{c^4 - c^3 + 1}{c^2 + 9}}$

(8) $\lim\limits_{x \to -4} (x+3)^{1998}$

$= (-4+3)^{1998}$

$= \boxed{1}$

(14) $\lim\limits_{x \to 2} \sqrt{x+3}$

$= \sqrt{2+3}$

$= \boxed{\sqrt{5}}$

(17) $\lim\limits_{x \to 0} \dfrac{|x|}{x} = \dfrac{0}{0}$

undefined @ $x = 0$
So can't use substitution

$\lim\limits_{x \to 0} \begin{cases} \frac{x}{x} = 1, & x > 0 \\ -\frac{x}{x} = -1, & x < 0 \end{cases}$

$\lim\limits_{x \to 0^+} \dfrac{|x|}{x} = 1$

$\lim\limits_{x \to 0^-} \dfrac{|x|}{x} = -1$

since $\lim\limits_{x \to 0^+} \dfrac{|x|}{x} \ne \lim\limits_{x \to 0^-} \dfrac{|x|}{x}$,

the limit does not exist.

(18) $\lim\limits_{x \to 0} \dfrac{(4+x)^2 - 16}{x} = \dfrac{0}{0}$

undefined @ $x = 0$,
So can't use substitution.

expand:

$\lim\limits_{x \to 0} \dfrac{\cancel{16} + 8x + x^2 - \cancel{16}}{x}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{x}(8 + x)}{\cancel{x}}$

$= \lim\limits_{x \to 0} 8 + x$

$= \boxed{8}$

(20) $\lim\limits_{t \to 2} \dfrac{t^2 - 3t + 2}{t^2 - 4} = \dfrac{0}{0}$

$= \lim\limits_{t \to 2} \dfrac{\cancel{(t-2)}(t-1)}{\cancel{(t-2)}(t+2)}$

$= \lim\limits_{t \to 2} \dfrac{t-1}{t+2}$

$= \boxed{\dfrac{1}{4}}$

(22) $\lim\limits_{x \to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x} = \dfrac{0}{0}$

$= \lim\limits_{x \to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x} \left(\dfrac{2(2+x)}{2(2+x)} \right)$

$= \lim\limits_{x \to 0} \dfrac{2 - (2+x)}{2x(2+x)}$

$= \lim\limits_{x \to 0} \dfrac{2 - 2 - x}{2x(2+x)}$

$= \lim\limits_{x \to 0} \dfrac{-\cancel{x}}{2\cancel{x}(2+x)}$

$= \lim\limits_{x \to 0} \dfrac{-1}{2(2+x)}$

$= \boxed{-\dfrac{1}{4}}$

(24) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x} = \dfrac{0}{0}$

$= \lim\limits_{x \to 0} \dfrac{\sin 2x}{x} \left(\dfrac{2}{2} \right)$

$= \lim\limits_{x \to 0} 2 \left(\dfrac{\sin 2x}{2x} \right)^{\nearrow 1}$

$= \lim\limits_{x \to 0} 2$

$= \boxed{2}$

(26) $\lim\limits_{x \to 0} \dfrac{x + \sin x}{x} = \dfrac{0}{0}$

$= \lim\limits_{x \to 0} \dfrac{x}{x} + \dfrac{\sin x}{x}^{\nearrow 1}$

$= \lim\limits_{x \to 0} 1 + 1$

$= \lim\limits_{x \to 0} 2$

$= \boxed{2}$

(28) $\lim\limits_{x \to 0} \dfrac{3 \sin 4x}{\sin 3x}$

$= \lim\limits_{x \to 0} 3 \cdot \dfrac{\sin 4x}{\sin 3x} \cdot \dfrac{3x}{3x} \cdot \dfrac{4x}{4x}$

$= \lim\limits_{x \to 0} 3 \cdot \dfrac{\sin 4x}{4x} \cdot \dfrac{3x}{\sin 3x} \cdot \dfrac{4x}{3x}$

$= \lim\limits_{x \to 0} 3 \cdot \dfrac{4}{3}$

$= \dfrac{4}{3} \cdot 3$

$= \boxed{4}$

(30) $\lim\limits_{x \to 2} \dfrac{x+1}{x^2 - 4} = \dfrac{3}{0}$

$\to \dfrac{\neq 0}{0}$ so VA

$\to$ Limit does not exist

(34) $\lim\limits_{x \to 2^-} \text{int } x$

$= \lim\limits_{x \to 2^-} [\![x]\!]$

(plug in 1.9999)

$\boxed{[\![1.99999]\!] = 1 \text{ So limit from left is one}}$

(36) $\lim\limits_{x \to 0^-} \dfrac{x}{|x|} = \dfrac{0}{0}$

(plug in -0.0001)

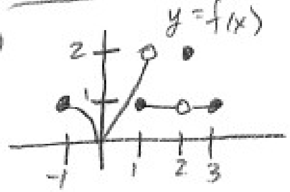
$\to \dfrac{-0.0001}{|-0.0001|}$

$= \dfrac{-0.0001}{0.0001}$

$= \boxed{-1}$

§2.1 cont

(38) $y=f(x)$



a) $\lim_{x\to 1^+} f(x) = 1$ (True)   b) $\lim_{x\to 2} f(x) = \text{DNE}$ (False, it is one.)

c) $\lim_{x\to 2} f(x) = 2$ (False)   d) $\lim_{x\to 1^-} f(x) = 2$ (True)

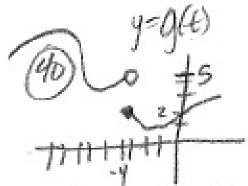
e) $\lim_{x\to 1^+} f(x) = 1$ (True)   f) $\lim_{x\to 1} f(x) = \text{DNE}$ (True)

g) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$ (True)

h) $\lim_{x\to c} f(x)$ exists at all $c \in (-1,1)$ (True)

i) $\lim_{x\to c} f(x)$ exists at all $c \in (1,3)$ (True)

(40) $y=g(t)$



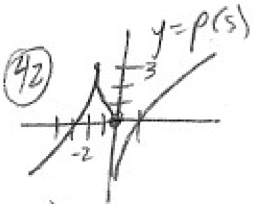
a) $\lim_{t\to -4^-} g(t) = 5$

b) $\lim_{t\to -4^+} g(t) = 2$

c) $\lim_{t\to -4} g(t) = \text{DNE}$

d) $g(-4) = 2$

(42) $y=p(s)$



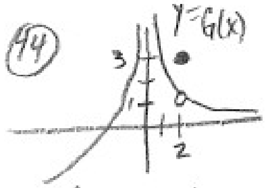
a) $\lim_{s\to -2^-} p(s) = 3$

b) $\lim_{s\to -2^+} p(s) = 3$

c) $\lim_{s\to -2} p(s) = 3$

d) $p(-2) = 3$

(44) $y=G(x)$



a) $\lim_{x\to 2^-} G(x) = 1$

b) $\lim_{x\to 2^+} G(x) = 1$

c) $\lim_{x\to 2} G(x) = 1$

d) $G(2) = 3$

(48) $Y_1 = \dfrac{x^2+x-2}{x+1}$

@ $x=1$, $Y_1 = \frac{0}{2} = 0$

$\boxed{\text{So it must be table a)}}$

(50) $\lim_{x\to b} f(x) = 7$, $\lim_{x\to b} g(x) = -3$

a) $\lim_{x\to b} (f(x)+g(x)) = 7-3 = \boxed{4}$

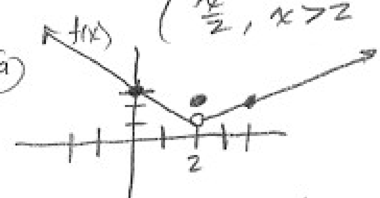
b) $\lim_{x\to b} f(x)\cdot g(x) = 7(-3) = \boxed{-21}$

c) $\lim_{x\to b} 4(g(x)) = 4(-3) = \boxed{-12}$

d) $\lim_{x\to b} \dfrac{f(x)}{g(x)} = \dfrac{7}{-3} = \boxed{-\frac{7}{3}}$

(52) $c=2$, $f(x) = \begin{cases} 3-x, & x<2 \\ 2, & x=2 \\ \frac{x}{2}, & x>2 \end{cases}$

(a)

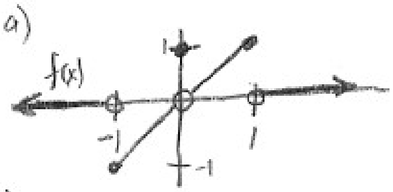


b) $\lim_{x\to 2^-} f(x) = 1 = \lim_{x\to 2^+} f(x)$

c) As a result of part b) the limit exists and is 1, $\lim_{x\to 2} f(x) = 1$

(58) $f(x) = \begin{cases} x, & -1 \le x < 0,\ 0 < x \le 1 \\ 1, & x = 0 \\ 0, & x < -1 \text{ or } x > 1 \end{cases}$

a)



b) $\lim_{x\to c} f(x)$ exists for all $c \neq -1, 1$. It does exist on the intervals $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

c) $\lim_{x\to c^-} f(x)$ only exists at no values

d) $\lim_{x\to c^+} f(x)$ only exists at no values.

(65) T or F

if $\lim_{x\to c^-} f(x) = 2$ and $\lim_{x\to c^+} f(x) = 2$, then $\lim_{x\to c} f(x) = 2$.

$\boxed{\text{TRUE, this is the definition of the Limit at } x=c.}$

(66) T or F

$\lim_{x\to 0} \dfrac{x+\sin x}{x} \stackrel{?}{=} 2$

$= \lim_{x\to 0} \dfrac{x}{x} + \dfrac{\sin x}{x}$

$= \lim_{x\to 0} 1+1 = 2 \rightarrow \boxed{\text{TRUE}}$

— Koji