§2.2 p. 76(2,3,7,12,16,20,30,34,36,42,54,56,58, AP1-4 Korpi period 8.91

② $f(x)=\frac{\sin 2x}{x}$

a) $\lim_{x\to\infty} f(x)=0$

b) $\lim_{x\to-\infty} f(x)=0$

c) HA @ $y=0$

③ $f(x)=\frac{e^{-x}}{x}=\frac{1}{xe^x}$

a) $\lim_{x\to\infty} f(x)=0$

b) $\lim_{x\to-\infty} f(x)=-\infty$

c) HA @ $y=0$

⑦ $f(x)=\frac{x}{|x|}$

a) $\lim_{x\to\infty} f(x)=1$

b) $\lim_{x\to-\infty} f(x)=-1$

c) HA @ $y=1$ & $y=-1$

⑫ $\lim_{x\to\infty}\frac{\sin(x^2)}{x}$

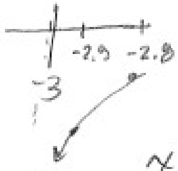
$=\lim_{x\to\infty}\left(\frac{1}{x}\right)\sin(x^2)$

$=0$

⑯ $\lim_{x\to-3^+}\frac{x}{x+3}=\frac{-3}{0}\to$ VA @ $x=-3$

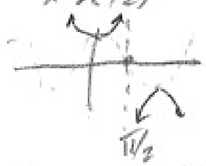
plug in $-2.9\to-29$

$-2.8\to-14$



So $\lim_{x\to-3^+}\frac{x}{x+3}=-\infty$

⑳ $\lim_{x\to(\pi/2)^+}\sec x$



From parent function knowledge..

$\lim_{x\to\pi/2^+}\sec x=-\infty$

㉚ $f(x)=\frac{1-x}{2x^2-5x-3}$

Denom $=0$

$(2x+1)(x-3)=0$

VA: $x=-\frac{1}{2}$, $x=3$

From calculator:

$\lim_{x\to-\frac{1}{2}^-} f(x)=\infty$ } $\lim_{x\to3^-} f(x)=\infty$

$\lim_{x\to-\frac{1}{2}^+} f(x)=-\infty$ } $\lim_{x\to3^+} f(x)=-\infty$

㉞ $f(x)=\frac{\cot x}{\cos x}=\frac{\cos x/\sin x}{\cos x}$

$=\frac{\cos x}{\cos x\cdot\sin x}$

VA when $\frac{\neq 0}{0}$, $\sin x=0$

when $x=\pi n$, $n\in\mathbb{Z}$

• for even π multiples, $x=c$

$\lim_{x\to c^-} f(x)=-\infty$

$\lim_{x\to c^+} f(x)=\infty$

• for odd π multiples, $x=d$

$\lim_{x\to d^-} f(x)=\infty$

$\lim_{x\to d^+} f(x)=-\infty$

㊱ $y=\frac{x^5-x^4+x+1}{2x^2+x-3}$

$\lim_{x\to\infty} y=\infty$, $\lim_{x\to-\infty} y=-\infty$

$\frac{x^5}{x^2}=x^3\to$ cubic asymptote

$\boxed{\text{choice c)}}$

㊷ $f(x)=\frac{3x^2-x+5}{x^2-4}$

$\lim_{x\to\infty} f(x)=3$, HA @ $y=3$

54 $f(x)=\begin{cases}\frac{x-2}{x-1}, & x\le 0\\ \frac{1}{x^2}, & x>0\end{cases}$

a) $\lim_{x\to-\infty} f(x)=1$

b) $\lim_{x\to\infty} f(x)=0$

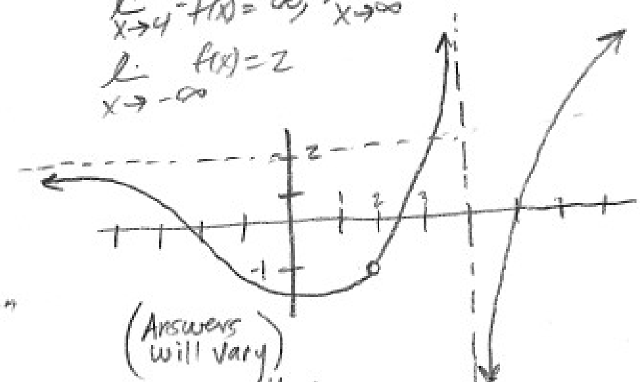
c) $\lim_{x\to0^-} f(x)=2$ (Direct sub into top piece)

d) $\lim_{x\to0^+} f(x)=\infty$ (parent function bottom piece)

56 $f(x)$: $\lim_{x\to2} f(x)=-1$, $\lim_{x\to4^+} f(x)=-\infty$

$\lim_{x\to4^-} f(x)=\infty$, $\lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to-\infty} f(x)=2$



(Answers will vary) come up with a different one, than mine!

58 $L\in\mathbb{R}$, $\lim_{x\to c} f(x)=L$

$\lim_{x\to c} g(x)=\infty$ or $-\infty$.

Can $\lim_{x\to c}(f(x)+g(x))$ be determined?

*Yes! from properties of limits,

$\lim_{x\to c}(f(x)+g(x))=\lim_{x\to c} f(x)+\lim_{x\to c} g(x)$

$=L+\infty$ or $L-\infty$

$=\infty$ or $-\infty$. the graph of $f(x)$ would only provide a vertical shift on $g(x)$ at $x=c$, which would not change the behavior of the y-values going to $\pm\infty$.

P1/

AP 1-4 (Next Page)

§2.2 cont

APQ

① $\lim_{x \to 3} \frac{x^2-x-6}{x-3} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}}$

$= \lim_{x \to 3} (x+2)$

$= \lim_{x \to 3} 5$

$= \boxed{5} \to$ (D)

② $\lim_{x \to 2^+} f(x) = ?$

$f(x) = \begin{cases} 3x+1, & x<2 \\ \frac{5}{x+1}, & x \geq 2 \end{cases}$

$\lim_{x \to 2^+} f(x) = \frac{5}{2+1} = \boxed{\frac{5}{3}} \to$ (A)

(direct sub into bottom piece)

③ $f(x) = \frac{3x^3 - x^2 + x - 7}{2x^3 + 4x - 5}$

HA@?

$\lim_{x \to \infty} f(x) = \frac{3x^3}{2x^3} = \frac{3}{2}$

$\boxed{\text{So HA @ } y = \frac{3}{2}} \to$ (E)

④ $f(x) = \frac{\cos x}{x}$

a) $\boxed{\text{Domain: } \{x \mid x \neq 0\} \text{ or } (-\infty, 0) \cup (0, \infty)}$ Division by zero

Range: $\frac{1}{x} \cos x = f(x)$

$\lim_{x \to \infty} f(x) = 0$ (Amplitude goes to zero)

$\lim_{x \to 0^+} f(x) = \infty$ and $-\infty$ (oscillates higher and lower, Amplitude approaches ∞)

so $\boxed{\text{Range: } \mathbb{R} \text{ or } (-\infty, \infty)}$

b) Even, odd, Neither?

Test: $f(-x) = \frac{\cos(-x)}{-x} = \frac{\cos x}{-x} = -\left(\frac{\cos x}{x}\right)$

$= -f(x) \to$ $\boxed{\text{Odd function with Origin Symmetry}}$

c) $\lim_{x \to \infty} f(x) = 0$

(from part a), the Amplitude goes to zero)

d) From sandwich Thm.

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\frac{1}{x}\right)(\cos x)$

$= \lim_{x \to \infty} \left(\frac{1}{x}\right) \cdot \lim_{x \to \infty} (\cos x)$

$= 0 \cdot 1$ or $0 \cdot (-1)$

$= 0$

— Koji