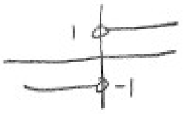
Cal AB/BC

§2.3 p.84 (7, 11-18, 20, 22, 24, 29, 40, 42, 47, 50, 51, 54, 55, 58, 59)

Korpi
period $\sqrt{-1}$

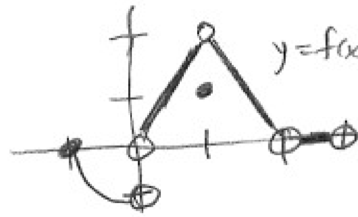
(7) $y=\frac{|x|}{x}=\begin{cases}\frac{x}{x}=1, & x>0\\ \frac{-x}{x}=-1, & x<0\end{cases}$



*jump discontinuity at $x=0$

11–18

$f(x)=\begin{cases}x^2-1 & ,-1\le x<0\\ 2x & ,0<x<1\\ 1 & ,x=1\\ -2x+4 & ,1<x<2\\ 0 & ,2<x<3\end{cases}$



(11) a) $f(-1)=0$ ✓

b) $\lim_{x\to -1^+} f(x)=0$ ✓

c) $\lim_{x\to -1^+} f(x)=0=f(-1)$ ✓

d) f is continuous at $x=-1$

(12) a) $f(1)=1$

b) $\lim_{x\to 1} f(x)=2$

c) $\lim_{x\to 1} f(x)\neq f(1)$

d) So $f(x)$ is NOT continuous @ $x=1$

(13) a) $f(2)=$ DNE

b) So $f(x)$ is NOT continuous at $x=2$

(14) f is continuous everywhere in $[-1,3)$ except at $x=0,1,2$

(15) for $f(x)$ to be continuous at $x=2$, $f(2)$ must equal $\lim_{x\to 2} f(x)=0$

(16) for $f(x)$ to be continuous at $x=1$, $f(1)$ must equal $\lim_{x\to 1} f(x)=2$

(17) $f(x)$ cannot be made continuous at $x=0$ since $\lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+} f(x)$ (Jump discon)

(18) $f(x)$ CAN be made continuous at 3 if $f(3)=\lim_{x\to 3^-} f(x)=0$

(20) $f(x)=\begin{cases}3-x & ,x<2\\ 2 & ,x=2\\ \frac{x}{2} & ,x>2\end{cases}$

*the 3 pieces are continuous everywhere, so we only check @ $x=2$

i) $f(2)=2$ ii) $\lim_{x\to 2^-} f(x)=3-2=1$

$\lim_{x\to 2^+} f(x)=\frac{2}{2}=1$ So $\lim_{x\to 2} f(x)=1$

iii) $f(2)=2\neq 1=\lim_{x\to 2} f(x)$

So there is a Removable pt. discon at $x=2$, Redefine $f(2)=\lim_{x\to 2} f(x)=1$

(22) $f(x)=\begin{cases}1-x^2 & ,x\neq -1\\ 2 & ,x=-1\end{cases}$

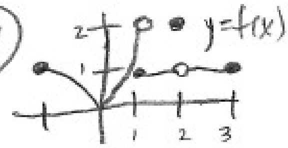
@ $x=-1$: i) $f(-1)=2$

ii) $\lim_{x\to -1} f(x)=1-(-1)^2=0$

iii) $f(-1)=2\neq 0=\lim_{x\to -1} f(x)$

So there is a Removable pt. discon @ $x=-1$, Redefine $f(-1)=\lim_{x\to -1} f(x)=0$

(24)



a) Non-Removable Jump Discon @ $x=1$

- Removable pt. discon @ $x=2$, Redefine $f(2)=\lim_{x\to 2} f(x)=1$

(29) $f(x)=\frac{x-4}{\sqrt{x}-2}$, not continuous at $x=4$

$f(x)=\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}=\frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}}=\boxed{\sqrt{x}+2}$

(40) $y=\begin{cases}\frac{x^2-1}{x-1}, & x\neq 1\\ 2, & x=1\end{cases}$

@ $x=1$: i) $y(1)=2$

ii) $\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}=2$

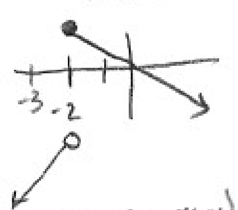
iii) $y(1)=2=\lim_{x\to 1} y$ so

y is continuous @ $x=1$

(42) $f(-2)$ exists

$\lim_{x\to -2^+} f(x)=f(-2)$

but $\lim_{x\to -2} f(x)=$ DNE



(Answers may vary)

(47) $f(x)=\begin{cases}x^2-1, & x<3\\ 2ax, & x\ge 3\end{cases}$

cont @ $x=3$:

i) $f(3)=2(a)(3)=6a$

ii) $\lim_{x\to 3^-} f(x)=3^2-1=8$

$\lim_{x\to 3^+} f(x)=6a$

So $\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$

$6a=8\to a=\frac{8}{6}=\boxed{\frac{4}{3}=a}$

Cal AB/BC
§2.3 cont

(50) $f(x)=\begin{cases} x^2+x+a, & x<1 \\ x^3, & x\geq 1 \end{cases}$

cont @ $x=1$:

i) $f(1)=1^3=1$

ii) $\lim_{x\to 1^-} f(x)=1^2+1+a=2+a$

$\lim_{x\to 1^+} f(x)=1^3=1$

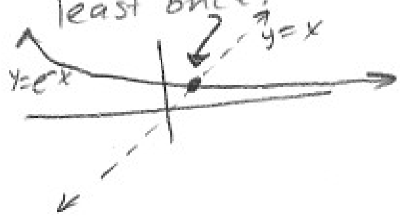
$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$

So $2+a=1$

$\boxed{a=-1}$

(51) $e^{-x}=x$ has at least one Solution because e^{-x} is exponential decay with y-int of (0,1) and HA @ $y=0$.

$y=x$ has y-int of (0,0) and increases W/o bound, they MUST intersect at least once.



(54) True (the book is incorrect)

(55) False (the book is incorrect)

see example #17 from this assigment

(58) $f(x)=\begin{cases} 2x, & 0<x<1 \\ 1, & x=1 \\ -x+3, & 1<x<2 \end{cases}$

which is false?

a) $f(1)=1 \rightarrow$ $\boxed{\text{FALSE}}$

b) $\lim_{x\to 0^+} f(x)=0 \rightarrow$ True

c) $\lim_{x\to 2^-} f(x)=1 \rightarrow$ True

d) $\lim_{x\to 1} f(x)=2 \rightarrow$ True

e) $\lim_{x\to 1} f(x)=2 \neq$

$f(1)=1 \rightarrow$ $\boxed{\text{FALSE}}$

(59) $f(x)=\dfrac{\cancel{x}\cancel{(x+1)}(x-2)^2\cancel{(x+1)^2}(x-3)^2}{\cancel{x}\cancel{(x+1)}\cancel{(x-2)}\cancel{(x+1)^2}(x-3)^3}$

$=\dfrac{x-2}{x-3}$ nonremovable (VA) $\boxed{@\ x=3}$

B/2

— Kopi