Cal AB/BC

§2.4 p.92 (2, 4, 8, 12, 14-16, 21, 26, 30-33, 35, 36, AP 1-4)

Korpi
Period $\sqrt{-1}$

(2) $f(x)=\sqrt{4x+1}$

a) $[0,2]$: $\dfrac{f(2)-f(0)}{2-0} = \dfrac{\sqrt{9}-\sqrt{1}}{2} = \dfrac{2}{2} = 1$

b) $[10,12]$: $\dfrac{f(12)-f(10)}{12-10} = \dfrac{\sqrt{49}-\sqrt{41}}{2} = \boxed{\dfrac{7-\sqrt{41}}{2}}$

(4) $f(x)=\ln x$

a) $[1,4]$: $\dfrac{f(4)-f(1)}{4-1} = \dfrac{\ln 4-\ln 1}{3} = \boxed{\dfrac{\ln 4}{3}}$

or $\frac{1}{3}\ln 4$ or $\ln 4^{1/3}$

or $\ln \sqrt[3]{4} \approx 0.462$

b) $[100,103]$: $\dfrac{f(103)-f(100)}{103-100} = \dfrac{\ln 103-\ln 100}{3} = \dfrac{\ln(103/100)}{3}$

$= \boxed{\frac{1}{3}\ln(1.03)} \approx 0.0099$

(8) Using $Q_1=(5,20)$, $Q_2=(7,38)$, $Q_3=(8.5,56)$, $Q_4=(9.5,72)$, $P=(10,80)$

a)

Secant Line	slope
PQ_1	$\frac{80-20}{10-5} = \frac{60}{5} = 12$
PQ_2	$\frac{80-38}{10-7} = \frac{42}{3} = 14$
PQ_3	$\frac{80-56}{10-8.5} = \frac{24}{1.5} = 16$
PQ_4	$\frac{80-72}{10-9.5} = \frac{8}{.5} = 16$

b) Speed at Point $P \approx 16$
(secant slopes are approaching this slope).

(12) $f(x) = y = x^2-3x-1$ @ $x=0$

a) slope @ $x=0$

$\lim\limits_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$\lim\limits_{h\to 0} \dfrac{(h^2-3h-1)-(-1)}{h}$

$\lim\limits_{h\to 0} \dfrac{h^2-3h}{h}$

$\lim\limits_{h\to 0} \dfrac{\cancel{h}(h-3)}{\cancel{h}}$

$0-3$

$\boxed{-3}$

(12b) eq. of tan line @ $x=0$ or $(0,-1)$

$y--1=-3(x-0)$

$y+1=-3x$

$\boxed{y=-3x-1}$

(12c) eq. of Normal line

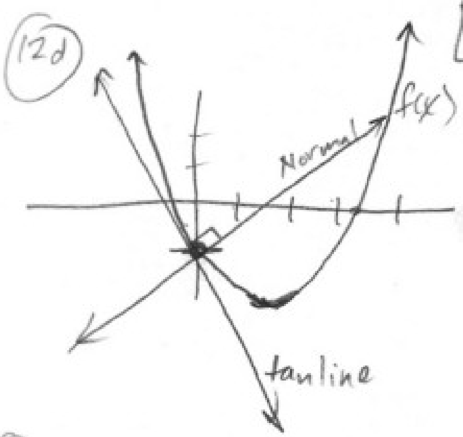
$m_\perp = \frac{1}{3}, P(0,-1)$

$y--1=\frac{1}{3}(x-0)$

$y+1=\frac{1}{3}x$

$\boxed{y=\frac{1}{3}x-1}$

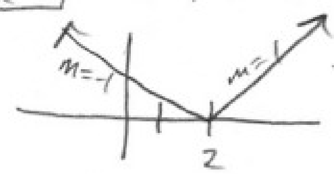
(12d)



(14) $f(x)=|x-2|$ @ $x=1$

find slope: $\boxed{\text{use: } |x|=\sqrt{x^2}}$

or graphically:



$\lim\limits_{h\to 0} \dfrac{|1+h-2|-|1-2|}{h}$

$=\lim\limits_{h\to 0} \dfrac{|h-1|-1}{h}$

$=\lim\limits_{h\to 0} \dfrac{\sqrt{(h-1)^2}-1}{h}\cdot\dfrac{\sqrt{(h-1)^2}+1}{\sqrt{(h-1)^2}+1}$

$=\lim\limits_{h\to 0} \dfrac{(h-1)^2-1}{h(|h-1|+1)}$

$=\lim\limits_{h\to 0} \dfrac{h^2-2h+1-1}{h(|h-1|+1)}$

$=\lim\limits_{h\to 0} \dfrac{\cancel{h}(h-2)}{\cancel{h}(|h-1|+1)}$

$=\dfrac{-2}{1+1}$

$=\dfrac{-2}{2}$

$=\boxed{-1}$

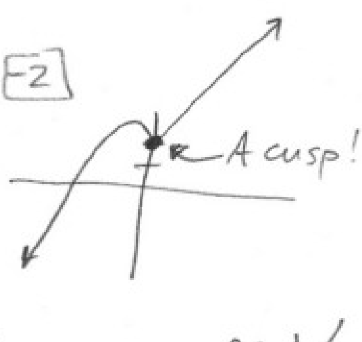
Slopes are $m=-1$ for $(-\infty,2)$

Slopes are $m=1$ for $(2,\infty)$

(15) $f(x)=\begin{cases} 2-2x-x^2, & x<0 \\ 2x+2, & x\geq 0 \end{cases}$ @ $x=0$

$\lim\limits_{h\to 0^-} \dfrac{[2-2(0+h)-(0+h)^2]-[2-0-0]}{h} = \lim\limits_{h\to 0^-} \dfrac{\cancel{2}-2h-h^2-\cancel{2}}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{h}(-2-h)}{\cancel{h}} = \boxed{-2}$

$\lim\limits_{h\to 0^+} \dfrac{[2(0+h)+2]-[2(0)+2]}{h} = \lim\limits_{h\to 0^+} \dfrac{2h+2-2}{h} = \lim\limits_{h\to 0^+} \dfrac{2h}{h} = \boxed{2}$



the two one-side limits of the difference quotients are different, so the tangent line does not exist at zero

Cal AB/BC
§2.4 continued

(16) $f(x) = \begin{cases} -x, & x<0 \\ x^2-x, & x \geq 0 \end{cases}$ @ $x=0$

$\lim_{h\to 0^-} \dfrac{[-(0+h)]-[-0]}{h} = \lim_{h\to 0^-} \dfrac{-h}{h} = \boxed{-1}$ ✓

$\lim_{h\to 0^+} \dfrac{((0+h)^2-(0+h)) - [0^2-0]}{h} = \lim_{h\to 0^+} \dfrac{h^2-h}{h}$

$= \lim_{h\to 0^+} \dfrac{\cancel{h}(h-1)}{\cancel{h}} = \boxed{-1}$ ✓

The tangent line DOES exist @ $x=0$ and it is -1.

(26) Volume of Sphere

$V = \frac{4}{3}\pi r^3$, Rate of change of Volume when $r=3$

$\lim_{h\to 0} \dfrac{\left[\frac{4}{3}\pi(2+h)^3\right]-\left[\frac{4}{3}\pi(2)^3\right]}{h}$

$= \lim_{h\to 0} \dfrac{\frac{4}{3}\pi[8+12h+6h^2+h^3]-\frac{4}{3}\pi(8)}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{\frac{32}{3}\pi} + 16\pi h + 8\pi h^2 + \frac{4}{3}\pi h^3 - \cancel{\frac{32}{3}\pi}}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{h}(16\pi + 8\pi h + \frac{4}{3}\pi h^2)}{\cancel{h}}$

$= \boxed{16\pi \text{ in}^3/\text{in}}$

(21) $y = \dfrac{1}{x-1}$, find slope at $x=a$

a) $\lim_{h\to 0} \dfrac{\dfrac{1}{(a+h)-1} - \dfrac{1}{a-1}}{h}\left(\dfrac{(a+h-1)(a-1)}{(a+h-1)(a-1)}\right)$

$= \lim_{h\to 0} \dfrac{(a-1)-(a+h-1)}{h(a+h-1)(a-1)}$

$= \lim_{h\to 0} \dfrac{a-1-a-h+1}{h(a+h-1)(a-1)}$

$= \lim_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(a+h-1)(a-1)}$

$= \boxed{\dfrac{-1}{(a-1)^2}}$

b) the slopes are Always Negative
the slopes get Smaller (tan line gets more horizontal) as x increases or decreases since denominator grows.

(30) $f(x) = 3-4x-x^2$ has horz. tan line when slope $=0$

$\lim_{h\to 0} \dfrac{[3-4(x+h)-(x+h)^2]-[3-4x-x^2]}{h} = 0$ Solve for x

$\lim_{h\to 0} \dfrac{\cancel{3}-\cancel{4x}-4h-\cancel{x^2}-2xh-h^2-\cancel{3}+\cancel{4x}+\cancel{x^2}}{h} = 0$

$\lim_{h\to 0} \dfrac{\cancel{h}(-4-2x-h)}{\cancel{h}} = 0$

$-4-2x=0$
$-4=2x$
$\boxed{x=-2}$

so $f(x)$ has a Horizontal tangent at $(-2, f(-2))$
$= \boxed{(-2,7)}$ → the Vertex

(31) $y = \dfrac{1}{(x-1)}$, where does slope $=-1$?

a) $\lim_{h\to 0} \dfrac{\dfrac{1}{x+h-1} - \dfrac{1}{x-1}}{h}\left(\dfrac{(x+h-1)(x-1)}{(x+h-1)(x-1)}\right) = -1$

$\lim_{h\to 0} \dfrac{(x-1)-(x+h-1)}{h(x+h-1)(x-1)} = -1$

$\lim_{h\to 0} \dfrac{\cancel{x}-\cancel{1}-\cancel{x}-h+\cancel{1}}{h(x+h-1)(x-1)} = -1$

$\lim_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(x+h-1)(x-1)} = -1$

$\dfrac{-1}{(x-1)^2} = -1$

$(x-1)^2 = 1$

$x-1=1$ or $x-1=-1$

$x=2$ or $x=0$

$(2,1)$ or $(0,-1)$

$\boxed{y-1=-(x-2)}$ $\boxed{y+1=-(x-0)}$

$y=-x+3$ $y=-x-1$

b) The Normal will have slope at $m=1$ at $x=0, x=2$ since 1 is the opp. recip. of -1.

@ $(2,1)$, $m=1$ at $(0,-1)$

$\boxed{y-1=(x-2)}$ $\boxed{y+1=(x-0)}$

$y=x-1$ $y=x-1$

Nice Prob

(X) (32) $y=9-x^2$, * find all tangents passing through $(1,12)$

the tangent lines (2 of them) must intersect the parabola

the slope function is

$m = \lim_{h\to 0} \dfrac{[9-(x+h)^2]-[9-x^2]}{h} = \boxed{-2x=m}$

gen eq for tan lines through $(1,12)$:

$y-12 = (-2x)(x-1)$

$y = -2x^2+2x+12$

set equal to $y=9-x^2$ → solve for x

$9-x^2 = -2x^2+2x+12$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3$, $x=-1$

$(3,0)$ $(-1,8)$

@ $(3,0)$, $m=-2(3)=-6$

$y-0=-6(x-3)$

$\boxed{y=-6x+18}$

@ $(-1,8)$, $m=(-2)(-1)=2$

$y-8=2(x+1)$

$\boxed{y=2x+10}$

§2.4 cont Cal

(33) \$billions / yr

a) Avg Rate of Spending [1990, 1995]

$= A = \frac{272.1 - 299.3}{1995 - 1990}$

$= \boxed{-5.4 \text{ billion dollars per yr}}$

b) Avg change [2000, 2001]

$A = \frac{305.5 - 294.5}{2001 - 2000}$

$= \boxed{11.0 \text{ billion dollars per year}}$

c) Avg change [2002, 2003]

$A = \frac{404.9 - 348.6}{2003 - 2002}$

$= \boxed{56.3 \text{ billion dollars per year}}$

d) e) f) skip g) the war in Iraq and increased spending to prevent terrorist attacks caused an unusual increase in defense spending.

(35) T or F:

If f(x) has a tangent line then there is a perpendicular line to it at this point called the Normal line

(36) False

$f(x) = |x|$ has No tangent line at $x = 0$ since there is a sharp turn there AND since the slopes are different from each side, $\lim_{x \to 0^-} f'(x) = -1$, $\lim_{x \to 0^+} f'(x) = 1$

AP (1) $f(x) = \sqrt{x+1}$, Avg. Rate of change on $(0,3)$

$A = \frac{f(3) - f(0)}{3 - 0} = \frac{2 - 1}{3} = \boxed{\frac{1}{3}}$

$\boxed{(D)}$

(2) which is false

$$f(x) = \begin{cases} \frac{3}{4}x, & 0 \le x < 4 \\ 2, & x = 4 \\ -x + 7, & 4 < x \le 6 \\ 1, & 6 < x < 8 \end{cases}$$

a) $\lim_{x \to 4^-} f(x) = \frac{3}{4}(4) = 3 \rightarrow T$

$\lim_{x \to 4^+} f(x) = -4 + 7 = 3$

b) $f(4) = 2 \rightarrow T$

c) $\lim_{x \to 6^-} f(x) = -6 + 7 = 1 \rightarrow T$

$\lim_{x \to 6^+} f(x) = 1$

d) $\lim_{x \to 8^-} f(x) = 1 \rightarrow T$

e) f is Not cont @ $x = 4$ since $\lim_{x \to 4} f(x) = 3 \ne 2 = f(4)$

$\boxed{(E)}$

(3) Tan line of $f(x) = 9 - x^2$ @ $x = 2$

$m = \lim_{h \to 0} \frac{[9 - (2+h)^2] - [9 - 2^2]}{h}$

$= \lim_{h \to 0} \frac{9 - 4 - 4h - h^2 - 5}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(-4 - h)}{\cancel{h}}$

$= -4 = m$, $f(2) = 9 - 4 = 5$

$y - 5 = -4(x - 2)$

$y = -4x + 8 + 5$

$y = -4x + 13$

$\boxed{(B)}$

F.R.

(4) $f(x) = 2x - x^2$

a) $f(3) = 2(3) - 3^2$

$= 6 - 9$

$= \boxed{-3}$

b) $f(3+h) = 2(3+h) - (3+h)^2$

$= 6 + 2h - 9 - 6h - h^2$

$= \boxed{-h^2 - 4h - 3}$

c) $\frac{f(3+h) - f(3)}{h} = \frac{(-h^2 - 4h - 3) - (-3)}{h} = \frac{\cancel{h}(-h - 4)}{\cancel{h}} = \boxed{-h - 4}$

d) Inst. Rate of change @ $x = 3$

$= \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} (-h - 4) = \boxed{-4}$

 -Korpi